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Abstract

Maximally parallel multiset rewriting systems (MPMRS) present a convenient way to express relations between unstructured objects. The functioning of various computational devices may be expressed in terms of MPMRS (e.g. register machines and many variants of P systems). In particular, this means that MPMRS are computationally complete; however, a direct translation leads to quite a big number of rules. Like for other classes of computationally complete devices, there is a challenge to find a universal system having the smallest number of rules. In this article we present different rule minimization strategies for MPMRS based on encodings and structural transformations. We apply these strategies to the translation of a small universal register machine (Korec, 1996) and we show that there exists a universal MPMRS with 23 rules. Since MPMRS are identical to a restricted variant of P systems with antiport rules, the results we obtained improve previously known results on the number of rules for that systems.

Keywords: Multiset rewriting, Universal computations, Small Register Machines, P Systems, Symport, Antiport

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1 Introduction

Multiset rewriting presents a convenient way to express chemical reactions. Indeed, there is a direct correspondence between chemicals, represented by multisets, and the reactions, represented by multiset rewriting. Some additional properties of the reactions' environment might be expressed by an additional control over the rewriting. This idea was heavily exploited and different multiset rewriting systems were proposed, we only mention here the Chemical Abstract Machine, CHAM, introduced in [3] and the Gamma language, first considered in [1] (see also a survey in [2]).

One of natural controls that can be added to the multiset rewriting is the maximal parallelism. This roughly corresponds to the idea of waiting until the chemical system reaches a stable state, *i.e.*, no more rules can be applied, for a particular step. More formally, during a rewriting step of a maximally parallel multiset rewriting system (MPMRS) all rules that can be applied should be applied.

MPMRS systems serve as a basis for P systems that were introduced by Gh. Păun in [8] as distributed parallel computing devices of biochemical inspiration. These systems are inspired from the structure and the functioning of a living cell. The cell is considered as a set of compartments (regions) nested one in another and which contain objects and evolution rules. Membranes are separators of regions; they may serve as communication channels between the regions. The basic framework specifies neither the nature of these objects, nor the nature of rules.

Numerous variants specify these two parameters by obtaining many different models of computing, see [12] for a comprehensive bibliography. One of these variants, P systems with *symport/antiport*, was introduced in [7]. This variant uses one of the most important properties of P systems: the communication. This property is so powerful, that it suffices by itself to reach the computational power of Turing machines only by moving objects between the regions. These systems have two types of rules: symport rules, when several objects go together from one region to another one, and antiport rules, when several objects from two regions are exchanged. In spite of a simple definition, they may compute all Turing computable sets of numbers, [7]. Several subsequent works have been dedicated to improve this result with respect to both the number of membranes used and the size of symport/antiport rules used inside the membranes. We refer to [10] for a survey of these investigations.

Since symport/antiport systems compute all recursively enumerable sets of numbers, it is possible to construct a universal symport/antiport P system, *i.e.*, a fixed system that will compute any partially recursive function if a corresponding input is provided. The article [4] constructs such a system having only 30 antiport rules. This result is based on a result from [5] where a universal register machine with 32 instructions is constructed.

Antiport P systems with one membrane (as considered in [4]) correspond in a direct manner to maximally parallel multiset rewriting systems (MPMRS). In

fact, any exchange rule $(u, out; v, in)$ of an antiport system becomes a multiset rewriting rule $u \rightarrow v$ and in both cases the application of rules is maximally parallel.

In this article we show that there is a universal MPMRS with 23 rules. Thus we improve the result from [4] and we obtain a universal antiport system with the same number of rules. This result is quite astonishing, because the machine from [5] that was the starting point of our construction has 25 computational branches. We also present different rule minimization strategies for MPMRS based on encodings and structural transformations. We also continue the discussion of the relation between the number of rules and their size started in [4].

2 Definitions

We recall here some basic notions of formal language theory we need in the rest of the paper. We refer to [11], [9] for further details.

We denote by $\mathbb{N}$ the set of all non-negative integers. A *multiset* S over O is a mapping $f_S : O \longrightarrow \mathbb{N}$. The mapping f_S specifies the number of occurrences of each element of S. The size of the multiset S is $|S| = \sum_{x \in O} f_S(x)$. An empty multiset is represented by λ.

We use the ordinary set notation in order to specify a multiset. In this case we either indicate the number of occurrences of each element as its power, or we give the mapping function f_S. For example the multiset containing 3 occurrences of element a, one occurrence of element b and zero occurrences of element c will be specified as $\{a^3, b\}$ or $a \rightarrow 3, b \rightarrow 1, c \rightarrow 0$. We shall also use a string notation in order to specify a multiset. In this case we write all elements of the multiset having a positive multiplicity in a string. For example, the previous multiset will be written as $aaab$ or a^3b.

The sum of two multisets P and Q over V, denoted by $P + Q$, is a multiset S such that $f_S(a) = f_P(a) + f_Q(a)$ for all a in V. Similarly, the difference of two multisets P and Q, denoted by $P - Q$, is a multiset S having $f_S(a) = f_P(a) \ominus f_Q(a)$ where $\ominus$ is the positive subtraction. A projection of a multiset X over a set O is denoted by $\pi_O(X)$.

2.1 Register machines

A deterministic *register machine* is the following construction:

$$M = (Q, R, q_0, q_f, P),$$

where Q is a set of states, $R = \{R_1, \ldots, R_k\}$ is the set of registers, $q_0 \in Q$ is the initial state, $q_f \in Q$ is the final state and P is a set of instructions (called also rules) of the following form:

1. $(p, [R_k P], q) \in P$, $p, q \in Q, p \neq q, R_k \in R$ (being in state p, increase register R_k and go to state q).

2. $(p, [R_k M], q) \in P$, $p, q \in Q, p \neq q, R_k \in R$ (being in state p, decrease register R_k and go to state q).

3. $(p, \langle R_k \rangle, q, s) \in P$, $p, q, s \in Q, R_k \in R$ (being in state p, go to q if register R_k is not zero or to s otherwise).

4. $(p, \langle R_k ZM \rangle, q, s) \in P$, $p, q, s \in Q, R_k \in R$ (being in state p, decrease register R_k and go to q if successful or to s otherwise).

5. $(q_f, STOP)$ (may be associated only to the final state q_f).

We note that for each state p there is only one instruction of the types above.

A configuration of a register machine is given by the $(k+1)$-tuple $(q, n_1, \cdots, n_k)$, where $q \in Q$ and $n_i \in \mathbb{N}, 1 \leq i \leq k$, describing the current state of the machine as well as the contents of all registers. A transition of the register machine consists in updating/checking the value of a register according to an instruction of one of types above and by changing the current state to another one. We say that the machine stops if it reaches the state q_f. We say that M computes a value $y \in \mathbb{N}$ on the *input* $x \in \mathbb{N}$ if, starting from the initial configuration $(q_0, x, 0, \cdots, 0)$, it reaches the final configuration $(q_f, y, 0, \cdots, 0)$.

It is well-known that register machines compute all partial recursive functions and only them, [6]. For every $n \in \mathbb{N}$, with every register machine M having n registers, an n-ary partial recursive function Φ_M^n is associated. Let $\Phi_0, \Phi_1, \Phi_2, \cdots$, be a fixed admissible enumeration of the set of unary partial recursive functions. Then, a register machine M is said to be *strongly universal* if there exists a recursive function g such that $\Phi_x(y) = \Phi_M^2(g(x), y)$ holds for all $x, y \in \mathbb{N}$.

We also note that the power and the efficiency of a register machine M depends on the set of instructions that are used. In [5] several sets of instructions are investigated. In particular, it is shown that there are strongly universal register machines with 22 instructions of form $[RiP]$ and $\langle RiZM \rangle$. Moreover, these machines can be effectively constructed.

Figure 1 shows this special universal register machine (more precisely in [5] only a machine with 32 instructions of type $[R_k P]$, $[R_k M]$ and $\langle R_k \rangle$ is constructed, and the machine below may be simply obtained from that one).

Here is the list of rules of this machine.

$(q_1, \langle R_1 ZM \rangle, q_3, q_6)$	$(q_3, [R_7 P], q_1)$	$(q_4, \langle R_5 ZM \rangle, q_6, q_7)$
$(q_6, [R_6 P], q_4)$	$(q_7, \langle R_6 ZM \rangle, q_9, q_4)$	$(q_9, [R_5 P], q_{10})$
$(q_{10}, \langle R_7 ZM \rangle, q_{12}, q_{13})$	$(q_{12}, [R_1 P], q_7)$	$(q_{13}, \langle R_6 ZM \rangle, q_{33}, q_1)$
$(q_{33}, [R_6 P], q_{14})$	$(q_{14}, \langle R_4 ZM \rangle, q_1, q_{16})$	$(q_{16}, \langle R_5 ZM \rangle, q_{18}, q_{23})$
$(q_{18}, \langle R_5 ZM \rangle, q_{20}, q_{27})$	$(q_{20}, \langle R_5 ZM \rangle, q_{22}, q_{30})$	$(q_{22}, [R_4 P], q_{16})$
$(q_{23}, \langle R_2 ZM \rangle, q_{32}, q_{25})$	$(q_{25}, \langle R_0 ZM \rangle, q_1, q_{32})$	$(q_{27}, \langle R_3 ZM \rangle, q_{32}, q_1)$
$(q_{29}, [R_0 P], q_1)$	$(q_{30}, [R_2 P], q_{31})$	$(q_{31}, [R_3 P], q_{32})$
$(q_{32}, \langle R_4 ZM \rangle, q_1, q_f)$		

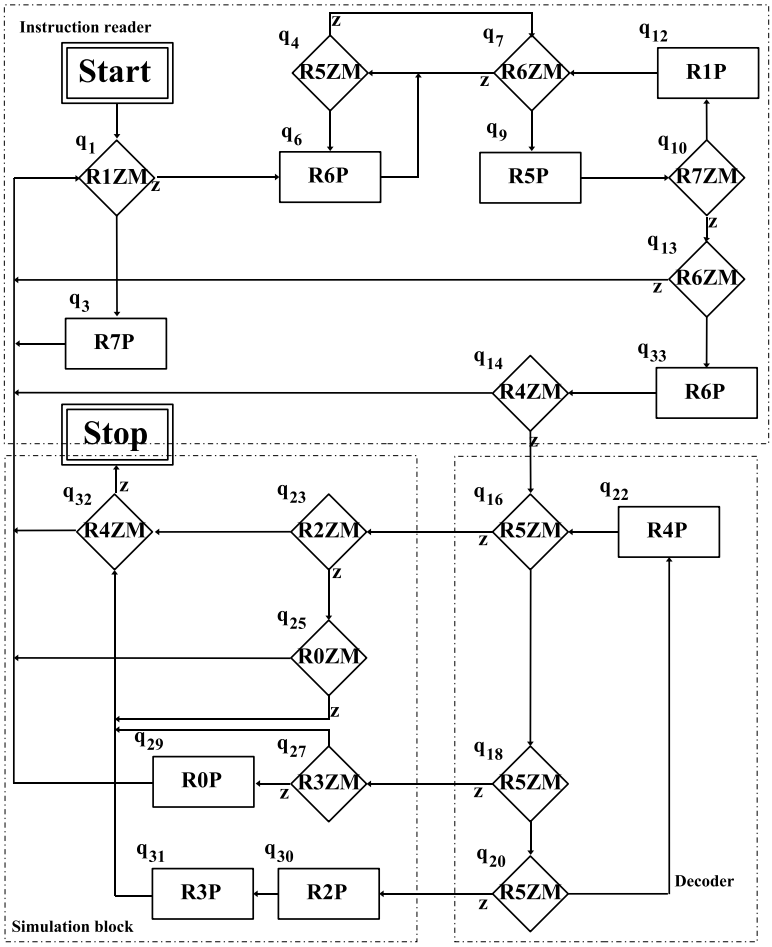


Figure 1: Flowchart of the strongly universal machine U_{22}

2.2 Maximally parallel multiset rewriting system

A *maximally parallel multiset rewriting system* (MPMRS) is the construct

$$\gamma = (O, \mathcal{I}, \mathcal{P}),$$

where O is an alphabet, $\mathcal{I}$ is the initial multiset and $\mathcal{P}$ is a set of multiset rewriting rules (productions) of form $u \to v$, $u \in O^+$, $v \in O^*$. We say that a rule $r \in \mathcal{P}$, $r : u \to v$ is *applicable* to a multiset $X \in O^+$ if $X \supseteq u$. Similarly, a set of rules $r_i : u_i \to v_i$, $1 \leq i \leq n$ is said to be applicable to X if $X \supseteq \sum_{1 \leq i \leq n} u_i$. We now define the *application* of a rule $r \in \mathcal{P}$ to a multiset $X \in O^+$ which produces a new multiset $Y \in O^*$; this is denoted by $X \xrightarrow{r} Y$. More exactly,

$$X \xrightarrow{r} Y \iff Y = X + u - v \text{ and } r \text{ is } u \to v.$$

A *maximally parallel transition*, written as $X \Rightarrow Y$, is performed if there are multisets $X_1, \cdots, X_{n-1}$, $n > 0$ such that $X \xrightarrow{r_1} X_1 \xrightarrow{r_2} X_2 \xrightarrow{r_3} \cdots \xrightarrow{r_{n-1}} X_{n-1} \xrightarrow{r_n} Y$ and $r_1, \cdots, r_n$ is a non-deterministically chosen maximally parallel set of rules applicable to X, *i.e.*, there is no $r \in \mathcal{P}$ such that $r, r_1, \cdots, r_n$ is applicable to X. In a more formal way,

- $\sum_{i=1}^{n} u_i \subseteq X$,
- $X - (\sum_{i=1}^{n} u_i) \not\supseteq u$ for all rules $(u \to v) \in \mathcal{P}$,
- $Y = (X - (\sum_{i=1}^{n} u_i)) + \sum_{i=1}^{n} v_i$.

The first condition indicates that these rules are applicable in parallel, *i.e.*, the the rules rewrite disjoint submultisets of X. The second condition is maximality: no other rule is applicable in parallel with them.

By $\Rightarrow^*$ we denote the reflexive and transitive closure of $\Rightarrow$.

We may perform a maximally parallel transition on a multiset $X \in O^*$ using the following algorithm.

Algorithm 1

Initially the list $\mathcal{L} \in \mathcal{P}^*$ is empty and the multiset X' over $O \cup \bar{O}$ is equal to X. Consider also the unmarking function $\mathfrak{u}(\bar{a}) = a$, $\mathfrak{u}(a) = a$, $a \in O$ and its extension to multisets. Consider $\bar{\mathcal{P}} = \{u \to \bar{v} \mid u \to v \in \mathcal{P}\}$ and suppose that rules from $\bar{\mathcal{P}}$ are ordered with respect to a total order $<$.

1. Take (non-deterministically) a rule $r \in \bar{\mathcal{P}}$ applicable to X' ($r : u \to \bar{v}$).
2. If the previous step was successful then update X' and $\mathcal{L}$: $X' = X' - u + \bar{v}$ and $\mathcal{L} = \mathcal{L}, r$. After that go to step 1.
3. If no rule in $\bar{\mathcal{P}}$ is applicable to X', inspect $\mathcal{L} = r_1, \cdots, r_l$, $l > 0$. If the rules appear in $\mathcal{L}$ according to the order $<$, *i.e.* $r_i \not< r_j$ when $i > j$, then put $Y = \mathfrak{u}(X')$ and return true.
4. Otherwise, fail and return false.

It is clear that $X \Rightarrow Y$ is a maximally parallel transition. We also define the set

$$NEXT(X) = \{Y \mid \text{ algorithm 1 returns true and } Y \text{ on input } X\}.$$

We define the set of *sentential forms* (called also *configurations*) $SF(\gamma)$ as

$$SF(\gamma) = \{w \mid \mathcal{I} \Rightarrow^* w\}.$$

We introduce the following additional notions. The *size* of a rule $u \to v$ is $|uv|$, *i.e.*, the size of the multiset uv. Let $\bar{O} = \{\bar{a} \mid a \in O\}$ be the set of marked symbols from O and $\mathfrak{u}$ be the unmarking morphism defined as in Algorithm 1. We say that a multiset X over $O \cup \bar{O}$ is stable if no rule can be applied to it: $u \not\subseteq X$ for all $(u \to v) \in \mathcal{P}$.

The result of the computation of γ is defined as

$$L(\gamma) = \{w \mid \mathcal{I} \Rightarrow^* w \text{ and } w \text{ is stable}\}.$$

2.3 State configurations

Now we distinguish an alphabet $R \subseteq O$ that we call the alphabet of *registers* or the *data* alphabet. A *state* configuration is the projection of a configuration over $O \setminus R$ (hence the registers alphabet is not a part of the state configuration). A state configuration B is *reachable* in one step from the state configuration A if there are multisets R', R'' over R such that there exists a maximally parallel transition $AR' \Rightarrow BR''$. We will denote this by $A \Rrightarrow B$. We remark that there might be several configurations reachable in one step from a particular configuration A. In the general case, the number of possible state configurations is not bounded, however we would like to consider MPMRS with a finite number of state configurations.

A *finite state maximally parallel multiset rewriting system* FsMPMRS is a tuple

$$\gamma = (O, R, R_t, \mathcal{I}, \mathcal{P}),$$

where $\gamma' = (O, \mathcal{I}, \mathcal{P})$ is a MPMRS which has a finite number of state configurations, *i.e.*, the projection of $SF(\gamma')$ over $O \setminus R$ is finite and $R_t \subseteq R \subsetneq O$ are the

terminal alphabet of registers and the alphabet of registers respectively. Moreover, we require that for any rule $r \in \mathcal{P}$, $r : u \to v$, u must contain at least one symbol from $O \setminus R$.

We would like to note that FsMPMRS correspond to the paradigm of the computation where the control (the program) is separated from the data. In our case the state configurations correspond to the program and the projection of a configuration to R corresponds to the data. Moreover, the restriction on the rules implies that the data cannot evolve by itself. This is a quite standard assumption. In particular, in case of register machines there is a strict separation between states and registers, and the registers cannot evolve by themselves.

The result of the computation of γ is the projection of $L(\gamma')$ over R_t:

$$L(\gamma) = \pi_{R_t}(\gamma').$$

We say that a rule $u \to v$ is a *pure state* rule if u contains no symbols from R, otherwise we call it a *register-dependent* rule.

The set of state configurations of a FsMPMRS may be computed iteratively as follows:

1. $C_0 = \{\mathcal{I}\}$.
2. $C_{i+1} = C_i \cup \pi_{O\setminus R}(Y)$, where $Y \in NEXT(X + R^\infty)$ for all $X \in C_i$.

We remark that $NEXT(X + R^\infty)$ is finite because there are no rules that involve only registers. In fact, $NEXT(X + R^\infty)$ may be obtained by computing $NEXT(X)$ for a system where register symbols in the rules are ignored. Moreover, we notice that actual infinity of register symbols is not needed, since the maximal number of them that can be consumed does not exceed the maximal number of register symbols that appear in the left hand side of a rule, multiplied by size of X. Therefore, when speaking about applicability of rules without worrying about the register symbols, we will write $X + R^\infty$, in the context of these observations.

2.3.1 Graphical notation.

We introduce a graphical notation for FsMPMRS. We represent a state configuration by a filled square. We also suppose that pure state rules precede register-dependent rules. Now, in order to represent the relations between state configurations we will depict the relation $\Rrightarrow$ by graphically representing the functioning of Algorithm 1. We take a state configuration X and apply the algorithm for the multiset $X + R^\infty$. It is clear that for any positive run of the algorithm that returns the multiset Y with $\mathcal{L} = r_1, \cdots, r_n$ the equation $X + R^\infty \overset{r_1}{\to} X_1 \overset{r_2}{\to} X_2 \overset{r_3}{\to} \cdots \overset{r_n}{\to} X_n$ holds, where $\mathfrak{u}(X_n) = Y$. More precisely, multisets X_i over $O \cup \bar{O}$, $1 \leq i \leq n$ are obtained in the second step of the algorithm. We take the projection over

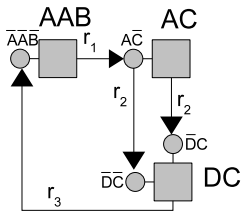
$O \cup \bar{O} \setminus (R \cup \bar{R})$ of each of these intermediate multisets and represent it by a circle. We also draw an arrow labelled by r_i between circles corresponding to $\pi_{O \cup \bar{O} \setminus (R \cup \bar{R})}(X_{i-1})$ and $\pi_{O \cup \bar{O} \setminus (R \cup \bar{R})}(X_i)$. Finally, we attach by a line the circle corresponding to a configuration Z to the square $\mathfrak{u}(Z)$. If all circles attached to a square represent multisets over $O \cup \bar{O}$ that are not stable with respect to pure state rules, such square is not filled.

The final diagram is obtained by repeating the above construction for all possible runs of the Algorithm 1 and for all state configurations. We recall that there is a finite number of state configurations and a finite number of possible runs of the Algorithm 1, hence the above process will stop at some moment.

Example 1 *Consider the following system* $\gamma = (\{A, B, C, D\}, \{E, F\}, \{F\}, \{AABEE\}, \mathcal{P})$*, where* $\mathcal{P}$ *contains the following rules:*

$$r_1 : AB \to C$$
$$r_2 : AE \to D$$
$$r_3 : DC \to AABF$$

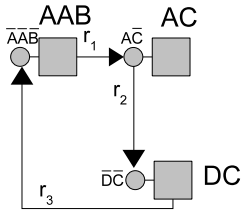
Clearly, the system γ *is a FsMPMRS that computes the language* $\{FF\}$*. Indeed, there are three state configurations* AAB*,* AC *and* CD *and there are no rules involving only* E *or* F *in the left-hand side. In a graphical way this system is represented as follows:*



The graphical notation described above not only describes the functioning of Algorithm 1, but also gives a tool which may be used for the description of the evolution of the system. Consider an arbitrary transition $X \Rightarrow Y$ corresponding to a parallel application of rules $r_1, \cdots, r_n$. If $X \xrightarrow{r_1} X_1 \xrightarrow{r_2} \cdots \xrightarrow{r_n} X_n = Y$, this can be graphically followed as a path from a square corresponding to $\pi_{O \setminus R}(X)$, going through circles corresponding to $\pi_{O \cup \bar{O} \setminus (R \cup \bar{R})}(X_i)$, $1 \leq i \leq n$, and the last one is attached to a square corresponding to $\pi_{O \setminus R}(Y)$. The maximality of parallelism should translate in the following way: no rules corresponding to arrows from the circle corresponding to $\pi_{O \cup \bar{O} \setminus (R \cup \bar{R})}(X_n)$ should be applicable to X_n.

Therefore, applying any maximally parallel transition to a configuration X means to start from the square $\pi_{O \setminus R}(X)$ and follow arrows to circles as long as possible, keeping track of symbols from R; when it is no longer possible, consider the square to which the last circle is attached.

Taking into account this description, we can simplify the diagram from the example above by the following observation. For any configuration X having $\pi_{O\setminus R}(X) = AAB$, if rule r_2 is not applicable from the circle $A\bar{C}$, then it will not be applicable from the square AC because otherwise it would be applied in the previous step. Hence, it may be eliminated:



This can be formalized as follows.

Proposition 1 *If the following conditions hold:*

- *Let A be a state configuration and B and C be two state configurations reachable in one step from A.*
- *There is a path (a sequence of rules) obtained by Algorithm 1 $A + R^\infty \rightarrow^* B' \stackrel{r_1}{\rightarrow} \cdots \stackrel{r_n}{\rightarrow} C'$ such that $\pi_{O\setminus R}(\mathfrak{u}(B')) = B$ and $\pi_{O\setminus R}(\mathfrak{u}(C')) = C$.*
- *There is no state configuration D other that those on the mentioned path, such that B is reachable in one step from D.*

Then the path labelled by $r_1, \cdots, r_n$ leading from B to C may never be involved in a computation and it may be eliminated from the diagram.

3 The Basic Simulation Technique

In this section we concentrate on a simple simulation of register machines by FsMPMRS. This simulation is done as follows. We represent a current configuration of a register machine M by a multiset (initially $\mathcal{I}$). In particular, the contents of a register $R_k \in R$ is represented by the number of symbols R_k which are present. The simulation of any incrementing or decrementing instruction of M is done by an appropriate set of rules.

In order to construct a MPMRS with a small number of rules we shall follow ideas presented in [4]. We take them as a starting point and after that we consider different minimization strategies that will decrease the number of used rules.

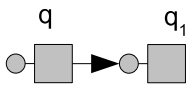
The system from [4] is based on a simulation of a special universal register machine U_{32} having 32 instructions taken from [5]. This construction may be rewritten in terms of $[R_iP]$ and $\langle R_iZM\rangle$ instructions, which gives 22 rules (9 incrementing instructions and 13 decrementing), see Figure 1.

The basic simulation strategy consists in a simulation of rules of an arbitrary register machine by multiset rewriting rules using the smallest number of the latter

ones. Any incrementing rule $(q, [R_k P], q_1)$ of register machine can be directly simulated by the rule

$$q \to R_k q_1, \tag{1}$$

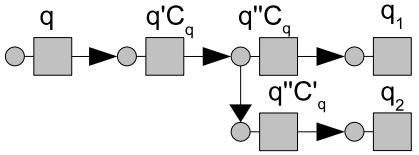
This corresponds to the following flowchart:



Any decrementing rule $(q, \langle R_{i_q} Z M \rangle, q_1, q_2)$ can be simulated using five rules:

$$\begin{array}{lll} q \to q' C_q, & & \\ q' \to q'', & C_q R_{i_q} \to C'_q, & \\ q'' C_q \to q_1, & q'' C'_q \to q_2 & \end{array} \tag{2}$$

This corresponds to the following flowchart:



This simulation is done as follows. Symbol q introduces symbols q' and C_q (the last one is called the *checker* for the state q).

After that symbol C_q tries to decrease register R_{i_q} and if it succeeds then it becomes C'_q. Now, depending on this information symbol q'', which replaced q', will choose the corresponding new state.

The choice between configurations $q'' C_q$ and $q'' C'_q$ depends on the presence of symbol R_{i_q}, *i.e.*, if register R_{i_q} is zero.

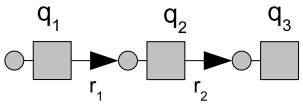
Applied to U_{32} this translation gives a FsMPMRS with 73 rules. We remark that these rules are of size at most 3. In the following sections we show different techniques which reduce the number of rules for the price of increasing their size.

4 Basic minimization strategies

In this section we present two basic minimization strategies. One of them is based on structural improvements and the other one is based on encodings. We present them in a general form and after that we show how they apply to the system that simulates U_{32}.

4.1 State Elimination

This minimization strategy performs an elimination of linear fragments in the flow-chart (by performing a kind of speed-up). Suppose that there are following two pure state rules, $r_1 = (q_1 \to q_2)$ and $r_2 = (q_2 \to q_3 R_k)$. This corresponds to the flowchart in the picture.

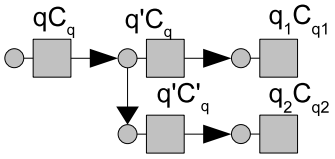


We observe that rules r_1 and r_2 may be combined and state q_2 may be eliminated by introducing a new rule $r = (q_1 \to q_3 R_k)$. In a similar way, any linear chain of pure state rules may be collapsed to a single rule (the size may be increased for each additional rule). We shall further refer to this technique as *intermediate state elimination.*

For U_{32} we observe that using intermediate state elimination technique we can reduce (2) to following rules (we also renamed q' by q):

$$\begin{aligned} &q \to q', && C_q R_{i_q} \to C'_q, \\ &q' C_q \to q_1 C_{q_1}, && q' C'_q \to q_2 C_{q_2} \end{aligned} \tag{3}$$

Graphically this can be represented as follows:



Moreover, we observe that for U_{32} in a most of the cases a decrementing instruction $(q, \langle R_k Z M \rangle, q_1, q_2)$ is followed by an incrementing instruction $(q_1, [R_{k_1} P], q_3)$ or $(q_2, [R_{k_2} P], q_4)$. Hence, one can simulate the incrementing instruction during the simulation of the previous decrementing instruction (by eliminating the unneeded state in between). For example, last two rules from (3) become

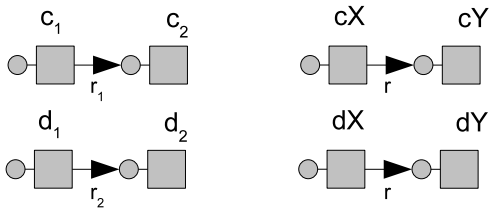
$$q' C_q \to q_3 R_{k_1} C_{q_3}, \qquad q' C'_q \to q_4 R_{k_2} C_{q_4}. \tag{4}$$

Of course, this increases the size of rules up to 5.

4.2 Gluing rules

In this section we shall consider techniques that will minimize the number of rules by performing more transitions between the configurations by fewer rules.

Informally, transitions $c_1 \stackrel{r1}{\to} c_2$ and $d_1 \stackrel{r2}{\to} d_2$ can be performed by the same rule $X \to Y$ if they are represented in a suitable way: $c_1 = cX$, $c_2 = cY$, $d_1 = dX$, $d_2 = cY$. In this case, we say that r_1 and r_2 may be *glued*. The following picture illustrates this:



In a more formal way one have to find a suitable encoding of state configurations such that:

- No state configuration is a submultiset of another state configuration.
- There should be at least 2 transitions that may be glued.

We would like to remark that it is only possible to glue transitions that increment registers equivalently, in particular, transitions that do not increment any register.

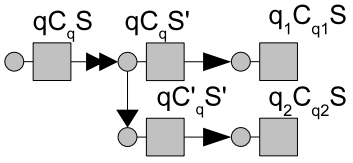
In what follows we apply the idea of gluing rules to the FsMPMRS system obtained by the basic simulation technique.

4.2.1 Phases

Consider now the rules (3). If we represent the state q by qS and the state q' by qS' then the first rule from (3) may be glued for all states q, *i.e.*, instead of $|Q|$ rules $q \to q'$ we obtain one rule $S \to S'$. We call the symbol S the *phase*, hence there will be two phases S and S'. The rules from (3) are replaced by:

$$\begin{aligned} &C_q R_{i_q} \to C'_q, && \\ &qS'C_q \to q_1 C_{q_1}, && qS'C'_q \to q_2 C_{q_2} \end{aligned} \tag{5}$$

Graphically this is represented at follows (where the double-headed arrow represents the rule $S \to S'$ common for all simulation blocks):



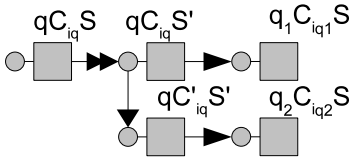
4.2.2 Independent Checkers

Another minimization idea comes from the observation that the information encoded in the checker C_q from (5) is redundant. If we take the set of first rules from (5) for all $q \in Q$, we observe that it is possible to glue rules that decrement the same register in the following way. We encode the sequence qC_q, respectively qC'_q, by symbols qC_{i_q}, respectively qC'_{i_q}, where i_q is the number of the register decreased by the instruction q of M. Now we may eliminate the first rule from (5) by introducing rules $C_i R_i \to C'_i$, $1 \leq i \leq |R|$. By convention, we will say that a state q of machine M is encoded by symbols $qC_{i_q}S$ and we will say that C_{i_q} is the checker for the state q. This transforms (5) into the following:

$$qS'C_q \to q_1 C_{q_1}, \qquad qS'C'_q \to q_2 C_{q_2}, \tag{6}$$

where $C_{i_{q1}}$ and $C_{i_{q2}}$ represent checkers for states q_1 and q_2. Of cause this introduces $|R|$ new rules, but finally we gain more because of the elimination of one rule for each $q \in Q$ from (5).

Graphically this is represented at follows:



4.2.3 Remarks

The improvements to U_{32} simulation presented above where implemented in [4], but they were classified in a different way. The cited article considers the relation between the size of rules and their number. In the table below we collect the results obtained by using the techniques above, as well as results from paper [4] which uses ideas similar to those used in the current and the next section (we underline these results):

Size	Number of rules
3	<u>73</u>
5	56
6	47
7	<u>43</u>
11	<u>30</u>

5 Further Minimization of U_{32} Simulation

In this section we show how to minimize the simulation of U_{32}. We start with the simulation using rules (6) and we do structural improvements based on some observations on the functioning of the system. After that we show how to glue most of the remaining rules by showing a suitable encoding of state configurations.

5.1 Structural improvements

The structural improvements presented in this section are in some sense a generalization of the intermediate state elimination technique.

5.1.1 Reducing decoder block

The first important improvement may be done by considering the decoder part of the machine from [5] (see the flowchart in Figure 1). In fact, this block does a division of R_5 by three. The result of this division is stored in register R_4 and according to the value of the remainder states q_{23}, q_{27} and q_{30} are chosen respectively. This behavior may be simulated by 5 rules which try to decrease register R_5 by 3 and make the choice depending on the result of this subtraction. The state q_{16} is now encoded by $q_{16}C_5C_5C_5S$.

$$\begin{aligned}
&C_5R_5 \to C_5', \\
&q_{16}C_5C_5C_5 \to q_{23}C_2, && q_{16}C_5C_5C_5' \to q_{27}C_3 \\
&q_{16}C_5C_5'C_5' \to q_{32}C_4R_3R_2, && q_{16}C_5'C_5'C_5' \to q_{16}C_5C_5C_5R_4
\end{aligned} \tag{7}$$

We note that we combined 2 addition instructions in the third branch using the elimination of intermediate states (q_{30} and q_{31}) by the mechanism discussed above. The subtraction by 3 is done using the maximal parallelism which permits to apply the rule $C_5R_5 \to C_5'$ three times if there are 3 copies of R_5. In [4] the idea of checking several registers at the same time is developed in more details, however here we will use another structural idea which is more efficient.

We integrally present the obtained flowchart in Figure 2. We use following conventions. The double-headed arrow represents the rule $S \to S'$ that changes the phase. Rules that decrement registers ($C_iR_i \to C_i'$) are represented by arrows starting with a perpendicular bar and labelled by $D0$–$D7$ enclosed in circle. Rules that increment registers are depicted by arrows with a dashed line and the incremented register(s) are depicted beside the line. These rules are labelled by a letter enclosed in a diamond. All other rules (which do not increment/decrement registers) are labelled by a number enclosed in a square.

5.1.2 State Elimination for Decrementing Rules

If we look at the flowchart in Figure 2 we observe that rules $D0$–$D3$ and $D7$ are used only once. This gives the possibility to combine these rules with rules that follow them using the state elimination technique. For example, the transformation done by rule 9 may be done directly in rule $D2$. However, in this case we reach the configuration $q_{32}C_4S'$ instead of $q_{32}C_4S$ because the rule $S \to S'$ is performed independently. We may solve this issue by introducing 3 phases instead of 2. In this case, phase 2 (marked by S') will be treated analogously to phase 1 (marked by S) and the move to the next state will be done in phase 3 (marked

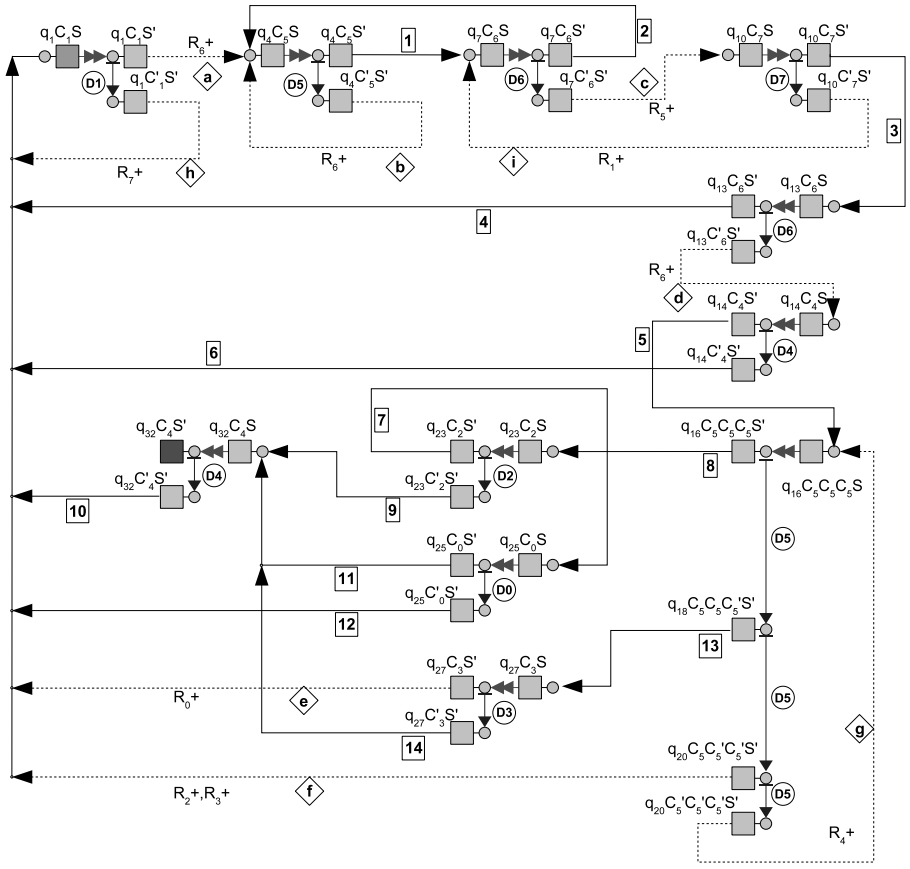


Figure 2: Multiset rewriting flowchart of U_{32} with improved decoder block

by S''). Moreover, the phase change may still be done by one rule. For this it is enough to replace S by XXX, S' by XXT and S'' by XTT and the rule $S \to S'$ by the rule $XX \to XT$. These changes permit to save 3 rules because rule $D1$ is a special case and it cannot be be combined with rule h. However, we can include the increment of R_7 in rule $D1$, in this case rule h becomes a non-incrementing rule and it will be labelled by $1a$.

The new flowchart is shown in Figure 3. We still depict phases by symbol S with primes because of the lack of the space. However, it is clear that the above substitution shall be done.

5.2 Encoding optimization

In this section we show how using the gluing minimization strategy the number of rules may be substantially decreased.

From Figure 3 we can see that transitions a, b and d may be potentially glued together as well as transitions labelled by numbers. All other rules are not eligible

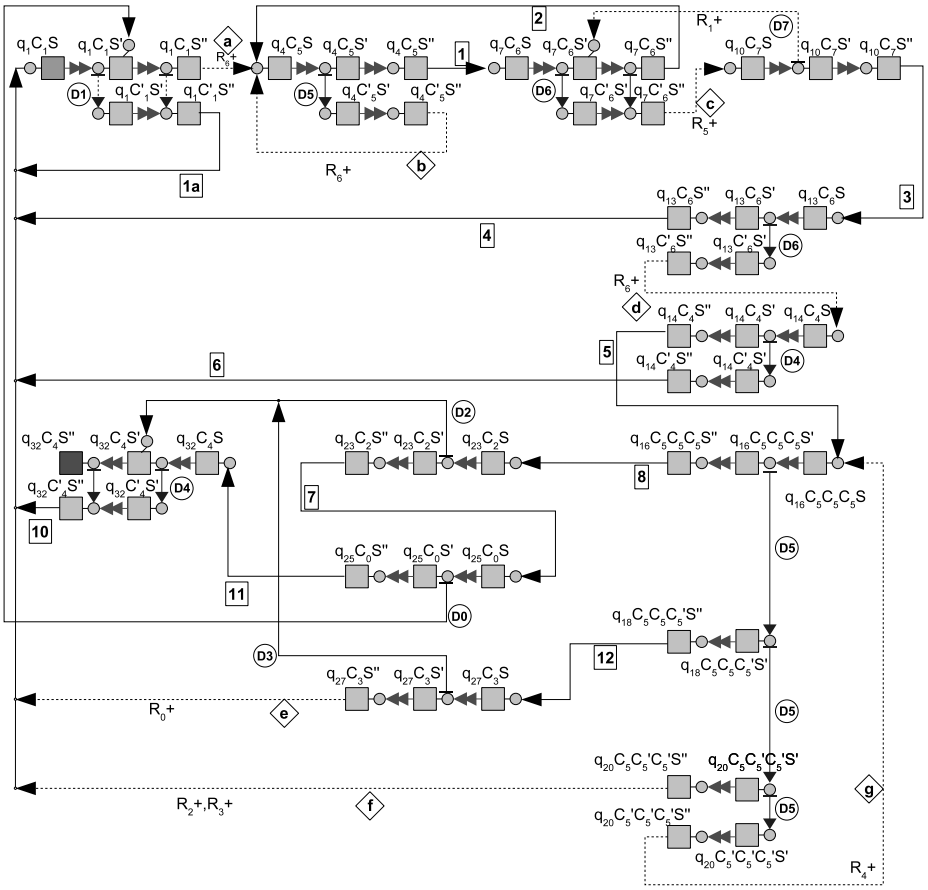


Figure 3: Multiset rewriting flowchart of U_{32} with 3 phases

for gluing. Consider the part of the flowchart that involves transitions labelled by numbers and all corresponding phases. Figure 4 depicts this. We denote rules that apply in parallel by drawing corresponding arrows beside each other and we do not show the S' phase (because in our case it differs from the S'' phase only by the phase symbol).

We remark that all rules labelled by numbers change the phase from S'' to S. We call such rules *phase changing* rules, in contrast to *non-phase* rules like $S \to S'$ (and $S' \to S''$) or $C_i R_i \to C'_i$. It is clear that non-phase rules may performed in parallel to each other or to a phase changing rule.

We found two strategies that permit to create suitable encodings in order to glue rules:

- End of phase discrimination.
- Maximally parallel unification.

The first strategy consists in assigning one phase changing rule per each arrow that enters a node with the maximal number of entering arrows. After that all

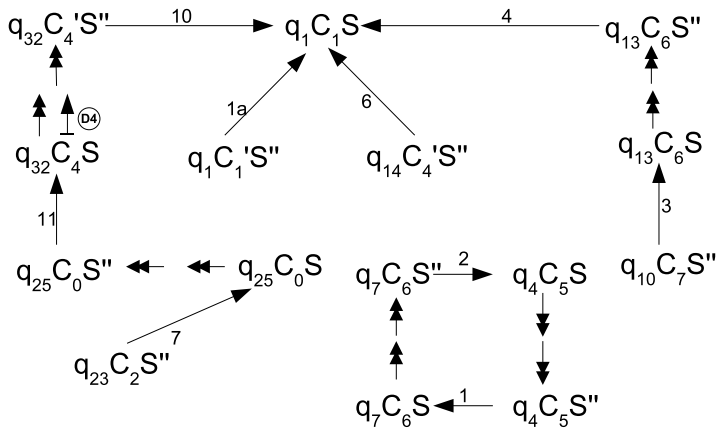


Figure 4: Part of the multiset rewriting flowchart of U_{32} showing only rules labelled by numbers.

other arrows are identified with one of the obtained arrows and the corresponding encoding is deduced.

The second technique permits to reduce the number of phase changing rules in the maximal node (and potentially in some other distant nodes). This technique is based on the fact that any number of phase changing rules that enter a node U may be reduced to only two rules, one phase changing and one non-phase rule. More precisely, we number rules from 0 to k, where k is the number of entering phase changing rules and denote by U_j, $0 \leq j \leq k$ corresponding nodes. After that we introduce new special symbols $A^j B^{k-j}$ in the encoding of U_j. We also consider that B^k is a part of the encoding of U. Such an encoding permits to make the transition from any of U_j, $0 \leq j \leq k$ to U by applying an appropriate phase changing rule and several parallel applications of the non-phase rule $A \to B$.

However, this technique has some limitations. In fact, it is applicable only if rules that are glued are preceded by rules that introduce the difference between encodings which is subject to the non-phase rule (the symbols A and B). In the case of our system, inherited from the basic simulation of U_{32}, this must be a rule that decrements a register. This is due to the fact that the non-phase rule may be applied in parallel to any other rule, so corresponding symbols must be introduced just before making the last step, namely during phase 2. If these symbols are introduced during phase 1, then in the case if corresponding computational branch is not chosen, for example if register is zero, then a wrong encoding (corresponding to some other state) is produced.

After applying all considerations above we found an encoding of the part of the flowchart represented in Figure 4 that permits to perform 9 rules (labelled by numbers) by only 3 new rules. This encoding is shown in Figure 5 and it is based on 2 phase changing rules $A = (IS'' \to JS)$ and $B = (JJMS'' \to JJNS)$ and

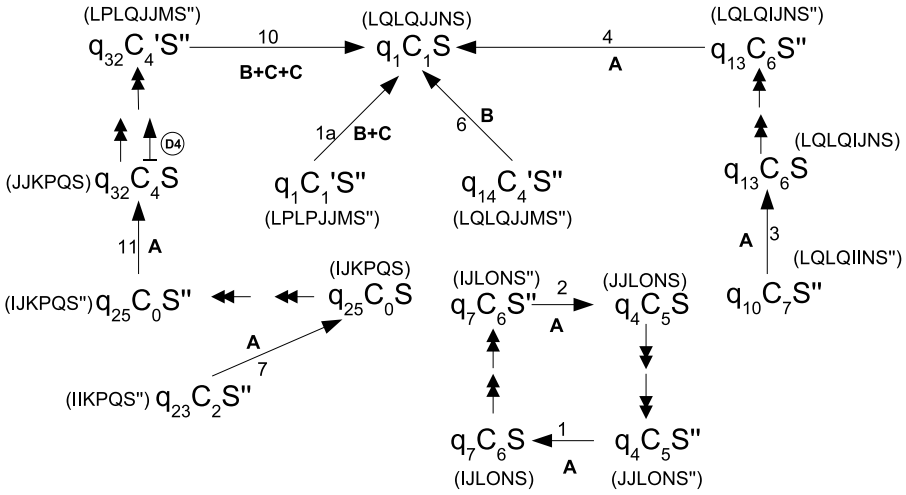


Figure 5: Part of the multiset rewriting flowchart of U_{32} showing only glued rules and the corresponding encoding.

on one non-phase rule $C = (LP \to LQ)$.

It was done as follows. The node q_1C_1S has the maximal number of entering arrows, hence we try to keep only the 4 rules that enter this node (10, 1a, 6 and 4) and express all other rules in their terms. However, before doing this it is better to apply the maximally parallel unification of the above 4 rules. We remark that rules 10, 1a and 6 exit from states reached by a decrement rule, while the rule 4 exits a state that is not reached by a decrement rule. Hence, we need two phase changing rules to perform rules 6 and 4. We denote them by B and A respectively. Rules 1a and 10 are performed by the combination $B + C$ and $B + C + C$ respectively. We recall that rule C is performed in parallel to any other rule.

Figure 6 shows the flowchart obtained after applying all ideas mentioned before. Arrows ending with a diamond correspond to rule C, while arrows ending with square correspond to rule A. The sparse arrow corresponds to rule B.

6 Formal Description of the System

In this section we give the formal description of the obtained system.

We constructed the system $\gamma = (O, R, \{R_1\}, \mathcal{I}, \mathcal{P})$, where

$$\begin{aligned} O &= R \cup \{C_3, C_5', C_6'\} \cup \{q_{16}, q_{27}\} \cup \{T, I, J, K, L, M, N, O, P, Q, T, X\}, \\ R &= \{R_i \mid 0 \le i \le 7\}, \\ \mathcal{I} &= LQLQJJNXXXR_0^{i_0} \cdots R_7^{i_7}. \end{aligned}$$

Here $i_0, \cdots, i_7$ is the contents of registers 0 to 7 of U_{32} and $LQLQJJNXXX$ is the encoding of the initial state q_1C_1S. The table below gives the set of rules $\mathcal{P}$.

$phase$	$XX \to XT$
$D0$	$IJKPQR_0 \to LQLQJJM$
$D1$	$LQLQJJNR_1 \to LPLPJJMR_7$
$D2$	$IIKPQR_2 \to JJKPQ$
$D3$	$q_{27}C_3R_3 \to JJKPQ$
$D4$	$JJKR_4 \to JJLLM$
$D5$	$JJOR_5 \to C'_5$
$D6$	$IJLR_6 \to C'_6$
$D7$	$IILQLQNR_7 \to IJLOR_1$
A	$ITT \to JXX$
B	$JJMTT \to JJNXX$
C	$LP \to LQ$
a	$LQLQJJNTT \to JJLOR_6XX$
b	$LC'_5TT \to JJLOR_6XX$
c	$OC'_6TT \to IILQLQNR_5XX$
d	$QLQNC'_6TT \to JJKQQR_6XX$
e	$q_{27}C_3TT \to LQLQJJNR_0XX$
f	$q_{16}JJOC'_5C'_5TT \to LQLQJJNR_2R_3XX$
g	$q_{16}C'_5C'_5C'_5TT \to q_{16}JJOJJOJJOXX$
5	$JJKQQTT \to q_{16}JJOJJOJJOXX$
8	$q_{16}JJOJJOJJOTT \to IIKPQMXX$
12	$q_{16}JJOJJOC'_5TT \to q_{27}C_3XX$

7 Conclusions

In this article we have investigated maximally parallel multiset rewriting systems (MPMRS) which correspond in a direct way to antiport P systems with one membrane. We constructed a universal MPMRS that computes any partially recursive function providing that the input is the encoding of a register machine computing the corresponding function as well as the value to be computed. Our construction uses 23 rules. This result is quite astonishing, because the machine from [5] that was the starting point of our construction uses 25 computational branches. This means that some branches in [5] do the same thing and are maybe redundant. Hence the result of this paper may possibly help to decrease the number of rules for universal register machines.

We also presented two different minimization strategies for MPMRS based on structural transformations (on the elimination of intermediate states) and on encodings (gluing rules) and discussed how their consequent application may decrease the number of rules in the simulation of U_{32}. These strategies may help to design more compact MPMRS and we leave the question open about a smaller number of rules for a universal MPMRS (or antiport P systems with one mem-

brane). Moreover, we observed a trade-off between the number of rules and their size (also observed in [4]) and we think that a further study of the relation between the size of rules and their influence on different minimization strategies (and of course their number) is interesting.

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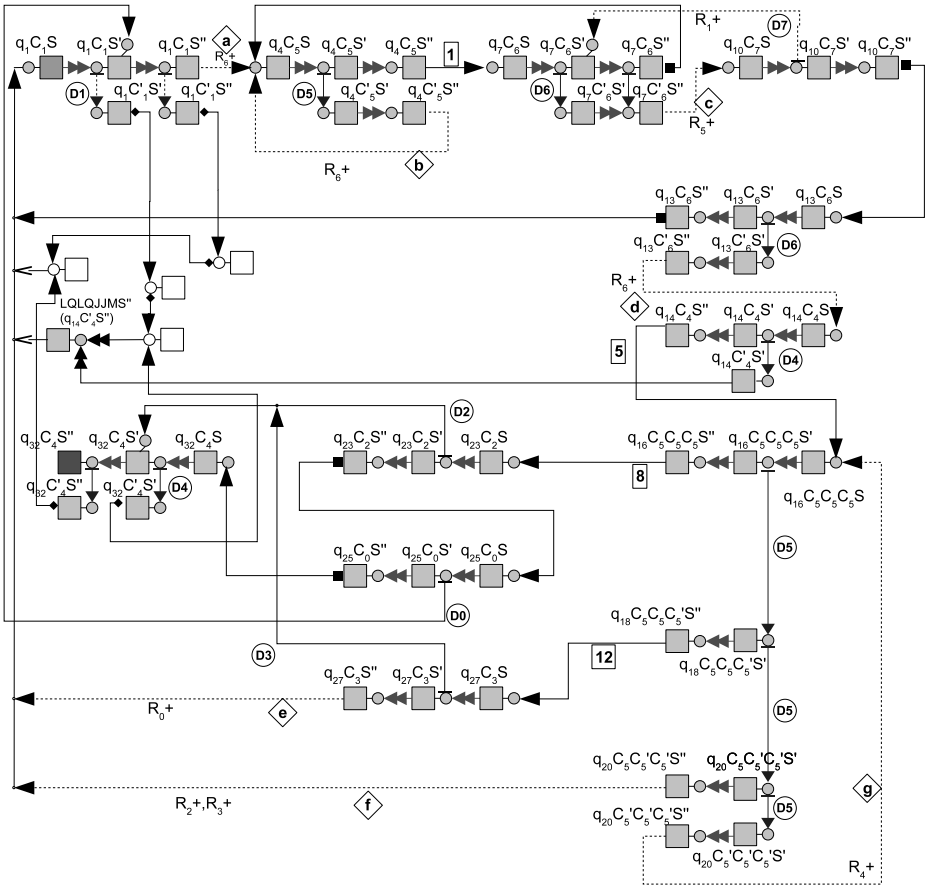

Figure 6: Multiset rewriting flowchart of U_{32} with glued rules.

References

[1] J.P. Banâtre, A. Coutant, D. Le Métayer, A parallel machine for multiset transformation and its programming style, Future Generation Computer Systems, 4 (1998), 133-144.

[2] J.P. Banâtre, P. Fradet, D. Le Métayer: Gamma and the Chemical Reaction Model: Fifteen Years After. In C.S. Calude, Gh. Păun, G. Rozenberg, A. Salomaa (Eds.), *Multiset Processing. Mathematical, Computer Science and Molecular Points of View. Lecture Notes in Computer Science*, **2235**, Springer, 2001, 17–44.

[3] G. Berry, G. Boudol, The chemical abstract machine, *Theoretical Computer Science* **96**, (1992), 217–248.

[4] E. Csuhaj-Varju, M. Margenstern, G. Vaszil, S. Verlan: Small Computationally Complete Symport/Antiport P systems, *Theoretical Computer Science* **372**(2-3), (2007) 152–164.

[5] I. Korec: Small universal register machines, *Theoretical Computer Science* **168**, (1996), 267–301.

[6] M. Minsky: *Computations: Finite and Infinite Machines*. Prentice Hall, Englewood Cliffts, NJ, 1967.

[7] A. Păun, Gh. Păun: The power of communication: P systems with symport/antiport. *New Generation Computing* **20**(3), (2002), 295–305.

[8] Gh. Păun: Computing with membranes. *Journal of Computer and System Sciences* **61**(1), (2000), 108–143.

[9] Gh. Păun: *Membrane Computing. An Introduction*. Springer-Verlag, 2002.

[10] Yu. Rogozhin, A. Alhazov, R. Freund: Computational power of symport/antiport: History, advances and open problems. In R. Freund, Gh. Păun, G. Rozenberg, A. Salomaa (Eds.), *Membrane Computing, 6th International Workshop, Vienna, Austria, July 18-21, 2005, Revised Selected and Invited Papers*, *Lecture Notes in Computer Science* **3850**, Springer (2006), 1–30.

[11] G. Rozenberg, A. Salomaa (Eds.): Handbook of Formal Languages. Vol. I-III., Springer, 1997.

[12] The P systems web page. `http://psystems.disco.unimib.it/`.

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