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Problem solving with multiple interdependent objectives

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Abstract

Decision making with interdependent multiple criteria is normal in standard business decision making; in mcdm theory the standard assumption is to assume that the criteria are independent, which makes optimal mcdm solutions less useful than they could be. In this paper we provide a short survey of methods that are both dealing with and making use of the interdependence of multiple criteria.

Keywords: Multiple criteria decision making, Fuzzy multiple objective programs

TUCS Laboratory

1 Introduction

There has been a growing interest and activity in the area of multiple criteria decision making (MCDM), especially in the last 35 years. Modeling and optimization methods have been developed in both crisp and fuzzy environments. The overwhelming majority of approaches for finding best compromise solutions to MCDM problems do not make use of the interdependences among the objectives. However, as has been pointed out by Carlsson [1], in modeling real world problems (especially in management sciences) we often encounter MCDM problems with interdependent objectives. In this paper we provide a short survey of methods (for a longer survey see [39]) that are both dealing with and making use of the interdependence of multiple criteria. Interdependence is a fairly obvious concept: consider a decision problem, in which we have to find a $x^* \in X$ such that three different criteria c_1, c_2 and c_3 all are satisfied, when c_1 and c_2 are *supportive* of each others, c_2 and c_3 are *conflicting*, and c_1 and c_3 again are *supportive* of each others (with respect to some directions). Unless it is obvious, the choice of an optimal decision alternative will become a very complex process with an increasing number of criteria.

2 Measures of interdependence

In this Section we review some measures of interdependences between the objectives, in order to provide for a better understanding of the decision problem, and to find effective and more correct solutions to multiple objective programming (MOP) problems. It is well known that there does not exist any concept of *optimal solution* universally accepted and valid for any multiobjective problem [37]. Delgado et al [32] provided a unified framework to use fuzzy sets and possibility theory in multicriteria decision and multiobjective programming. Felix [33] presented a novel theory for multiple attribute decision making based on fuzzy relations between objectives, in which the interactive structure of objectives is inferred and represented explicitely. Carlsson [1] used the fuzzy Pareto optimal set of nondominated alternatives as a basis for an OWA-type operator [36] for finding a best compromise solution to MCDM problems with interdependent criteria.

Combining the results of [1, 2, 32, 33, 37, 36, 38] we provided a new method for finding a compromise solution to fuzzy and crisp multiple objective programming problems in [3, 4, 6, 7] by using explicitely the interdependences among the objectives.

In multiple objective programs, application functions are established to measure the degree of fulfillment of the decision maker's requirements (achievement of goals, nearness to an ideal point, satisfaction, etc.) about the objective functions (see e.g. [32, 38]) and extensively used in the process of finding "good compromise" solutions. In [3] generalizing the principle of application function to fuzzy multiple objective programs (FMOP) with interdependent objectives, we defined a large family of application functions for FMOP in order to provide for a better understanding of the decision problem, and to find effective and more correct solutions. In [5] we defined interdependencies among the objectives of FMOP by using fuzzy if-then rules. In [6] we demonstrated that the use of interdependences among objectives of a MOP in the definition of the application functions provides for more correct solutions and faster convergence.

Example 2.1. *Consider the following problem with multiple objectives*

$$\max_{x \in X} \{f_1(x), \dots, f_k(x)\} \tag{1}$$

where $f_i(x) = \langle c_i, x \rangle = c_{i1}x_1 + \cdots + c_{in}x_n$ *and* $||c_i|| = 1,\ i = 1, \dots, k.$

Definition 2.1. *[6] Let* $f_i(x) = \langle c_i, x \rangle$ *and* $f_j(x) = \langle c_j, x \rangle$ *be two objective functions of (1). Then the measure of conflict between* f_i *and* f_j*, denoted by* $\kappa(f_i, f_j)$*, is defined by*

$$\kappa(f_i, f_j) = \frac{1 - \langle c_i, c_j \rangle}{2}.$$

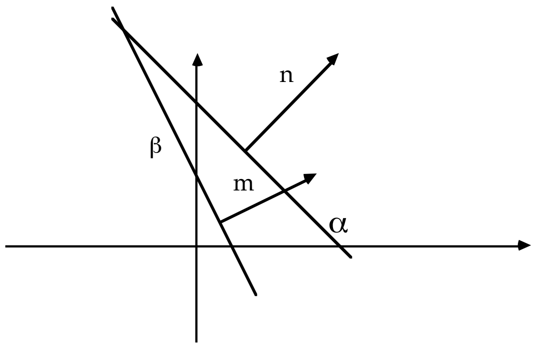

Figure 1: The measure of conflict between α and β is calculated from $\langle \mathbf{n}, \mathbf{m} \rangle$.

We illustrate the meaning of the measure of conflict by a biobjective two-dimensional decision problem

$$\max_{x \in X} \{\alpha(x), \beta(x)\}$$

where $\alpha(x) = \langle \mathbf{n}, x \rangle$ *and* $\beta(x) = \langle \mathbf{m}, x \rangle$*. The bigger the angle between the lines* α *and* β *the bigger the degree of conflict between them.*
If $\kappa(\alpha, \beta) = 1/2$ *and the set of feasible solutions is a convex polyhedron in* $\mathbb{R}^n$ *then* α *and* β *attend their independent maximum at neighbour vertexes of* X*. It is clear that we we increase the first objective function in its gradient direction then the value of the second objective function will not be affected (and vica versa). It is why we say that in this case* α *and* β *are independent in their gradient directions.*
If $\kappa(\alpha, \beta) = 0$ *and the set of feasible solutions is a convex polyhedron subset of* $\mathbb{R}^n$ *then* α *and* β *attend their independent maximum at the same vertex of* X*.*

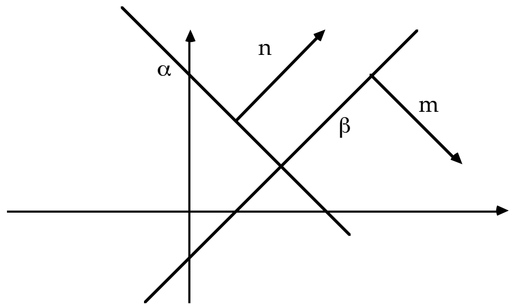

Figure 2: $\kappa(\alpha, \beta) = 1/2$ - the case of perpendicular objectives.

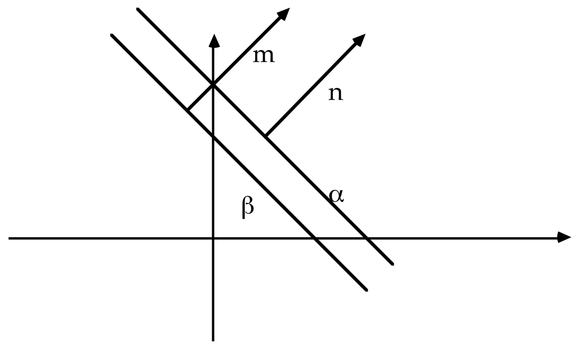

Figure 3: $\kappa(\alpha, \beta) = 0$ - the case of parallel objectives.

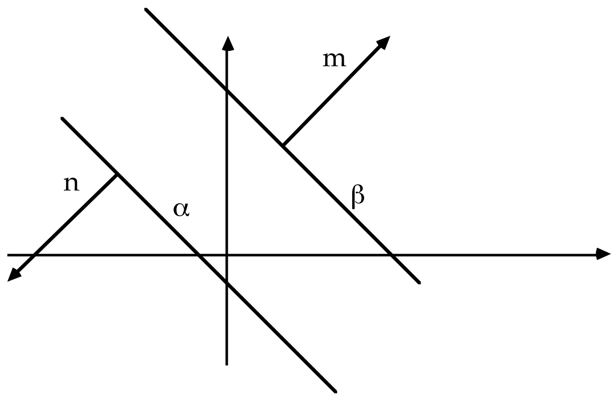

Figure 4: $\kappa(\alpha, \beta) = 1$ - the case of opposite objectives.

Definition 2.2. *The complexity of the problem (1) is defined as*

$$\Omega = \frac{\sum_{i,j}^{k} \kappa(f_i, f_j)}{2}.$$

It is clear that $\Omega = 0$ iff all the objectives are parallel, i.e. we have a single objective problem.

These principles have appeared to be useful in the construction of a scalarizing function, when we search for a nondominated solution being closest to an ideal point in a given metric.

In [7, 8] we analyzed multiple objective programming problems with additive interdependences. In [9, 10, 13, 14, 15, 16] we considered multiple objective programming problems with compound interdependences, i.e. the case when the states of some chosen objective are attained through supportive or inhibitory feedbacks from several other objectives. MOP problems with independent objectives (i.e. when the cause-effect relations between the decision variables and the objectives are completely known) will be treated as special cases of the MOP in which we have interdependent objectives. In [12] we considered biobjective decision problems with interdependent objectives. First we stated biobjective decision problems with independent objectives and by introducing additive linear interdependences between the objective functions we explain the behavior of compromise solutions.

3 Linear Interdependences in MOP

A typical statement of an independent MOP is

$$\max_{x \in X} \{f_1(x), \dots, f_k(x)\} \tag{2}$$

where f_i is the i-th objective function, x is the decision variable, and X is a subset, usually defined by functional inequalities. However, as has been shown by Felix [40] there are management issues and negotiation problems, in which one often encounters the necessity to formulate MOP models with interdependent objective functions, in such a way that the values of the objective functions are determined not only by the decision variables but also by the values of one or more other objective functions. Suppose now that the objectives of (2) are interdependent, and the value of an objective function is determined by a linear combination of the values of other objectives functions. That is

$$f_i'(x) = f_i(x) + \sum_{j=1,\ j \neq i}^{k} \alpha_{ij} f_j(x),\ 1 \leq i \leq k \tag{3}$$

or, in matrix form

$$\begin{pmatrix} f_1'(x) \\ f_2'(x) \\ \vdots \\ f_3'(x) \end{pmatrix} = \begin{pmatrix} 1 & \alpha_{12} & \dots & \alpha_{1k} \\ \alpha_{21} & 1 & \dots & \alpha_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{k1} & \alpha_{k2} & \dots & 1 \end{pmatrix} \begin{pmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_k(x) \end{pmatrix}$$

where α_{ij} is a real numbers denoting the grade of interdependency between f_i and f_j: If $\alpha_{ij} > 0$ then we say that f_i is supported by f_j; if $\alpha_{ij} < 0$ then we say that f_i is hindered by f_j; if $\alpha_{ij} = 0$ then we say that f_i is independent from f_j.
In such cases, i.e. when the feed-backs from the objectives are directly proportional to their independent values, then we say that the objectives are *linearly* interdependent. The matrix of interdependences, (α_{ij}), denoted by $I(f_1, \dots, f_k)$, is called the interdependency matrix of (2). It is clear that if $\alpha_{ij} = 0, \ \forall i \neq j$, i.e.

$$I(f_1, \dots, f_k) = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

then we have an MOP problem with independent objective functions.
To explain the issue more exactly, consider a three-objective problem with linearly interdependent objective functions

$$\max_{x \in X} \{ f_1(x), f_2(x), f_3(x) \} \tag{4}$$

Taking into consideration that the objectives are linearly interdependent, the interdependent values of the objectives can be expressed by

$$\begin{aligned} f_1'(x) &= f_1(x) + \alpha_{12} f_2(x) + \alpha_{13} f_3(x) \\ f_2'(x) &= f_2(x) + \alpha_{21} f_1(x) + \alpha_{23} f_3(x) \\ f_3'(x) &= f_3(x) + \alpha_{31} f_1(x) + \alpha_{32} f_2(x) \end{aligned}$$

That is

$$\begin{pmatrix} f_1'(x) \\ f_2'(x) \\ f_3'(x) \end{pmatrix} = \begin{pmatrix} 1 & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & 1 & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & 1 \end{pmatrix} \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{pmatrix}$$

For example, depending on the values of α_{ij} we can have the following simple linear interdependences among the objectives of (4)

- if $\alpha_{12} = 0$ then we say that f_1 is independent from f_2;
- if $\alpha_{12} > 0$ then we say that f_2 unilaterally supports f_1;
- if if $\alpha_{12} < 0$ then we say that f_2 hinders f_1;
- if $\alpha_{12} > 0$ and $\alpha_{21} > 0$ then we say that f_1 and f_2 mutually support each others;
- if $\alpha_{12} < 0$ and $\alpha_{21} < 0$ then we say that f_1 and f_2 are conflicting;
- if $\alpha_{12} + \alpha_{21} = 0$ then we say that f_1 are f_2 are in a trade-off relation.

It is clear, for example, that if f_2 unilaterally supports f_1 then the bigger the value of f_2 (supporting objective function) the bigger its support to f_1 (supported objective function).

Example 3.1. *To illustrate our ideas consider the following simple decision problem.*

$$\max\{x, 1-x\}, \textit{subject to } x \in [0,1] \tag{5}$$

Choosing the minimum-norm to aggregate the values of objective functions this problem has a unique solution $x^* = 1/2$ *and the optimal values of the objective functions are* $(0.500, 0.500)$. *Suppose that for example* f_1 *is unilaterally supported by* f_2 *on the whole decision space* $[0,1]$ *and the degree of support is given by*

$$f_1'(x) = f_1(x) + 1/2f_2(x) = x + 1/2(1-x) = 1/2 + x/2$$

Then (5) turns into the following problem

$$\max\{1/2 + x/2, 1-x\}$$
$$x \in [0,1]$$

Choosing the minimum-norm to aggregate the values of objective functions this problem has a unique solution $x^* = 1/3$ *and the optimal values of the objective functions are* $(0.667, 0.667)$.

4 Optimization under fuzzy if-then rules

Suppose we are given a mathematical programming problem in which the functional relationship between the decision variables and the objective function is not completely known. Our knowledge-base consists of a block of fuzzy if-then rules, where the antecedent part of the rules contains some linguistic values of the decision variables, and the consequence part consists of a linguistic value of the objective function. Between 1998 and 2003 we introduced [17, 18, 19, 20] a novel statement of (multiple objective) fuzzy mathematical programming problems and provided a method for findig a fair solution to these problems, namely we suggested the use of Tsukamoto's fuzzy reasoning method to determine the crisp functional relationship between the objective function and the decision variables, and solve the resulting (usually nonlinear) programming problem to find a fair optimal solution to the original fuzzy problem.
The rules represent our knowledge-base for the fuzzy optimization problem. The fuzzy partitions for lingusitic variables will not ususally satisfy ε-completeness, normality and convexity. In many cases we have only a few (and contradictory) rules. Therefore, we can not make any preselection procedure to remove the rules which *do not play any role* in the optimization problem. All rules should be considered when we derive the crisp values of the objective function.

We have chosen Tsukamoto's fuzzy reasoning scheme, because the individual rule outputs are crisp numbers, and therefore, the functional relationship between the input vector y and the system output $f(y)$ can be relatively easily identified (the only thing we have to do is to perform inversion operations).

Example 4.1. *Consider the optimization problem [19]*

$$\min f(x); \text{ subject to } \{x_1 + x_2 = 1/2, \ 0 \le x_1, x_2 \le 1\}, \tag{6}$$

and $f(x)$ is given linguistically as

$$\Re_1(x): \quad \textit{if } x_1 \textit{ is } small \textit{ and } x_2 \textit{ is } small \textit{ then } f(x) \textit{ is } small,$$
$$\Re_2(x): \quad \textit{if } x_1 \textit{ is } small \textit{ and } x_2 \textit{ is } big \textit{ then } f(x) \textit{ is } big,$$

and the universe of discourse for the linguistic value of f is also the unit interval $[0,1]$. We will compute the firing levels of the rules by the product t-norm. Let the membership functions in the rule-base $\Re$ be defined by

$$\text{small}(t) = 1 - t, \quad \text{big}(t) = t, \ t \in [0,1].$$

Let (y_1, y_2) be an input vector to the fuzzy system. Then the firing levels of the rules are

$$\alpha_1 = \text{small}(y_1) \times \text{small}(y_2) = (1 - y_1)(1 - y_2),$$
$$\alpha_2 = \text{small}(y_1) \times \text{big}(y_2) = (1 - y_1)y_2,$$

It is clear that if $y_1 = 1$ then no rule applies because $\alpha_1 = \alpha_2 = 0$. So we can exclude the value $y_1 = 1$ from the set of feasible solutions. The individual rule outputs are (see Figure 5)

$$z_1 = 1 - (1 - y_1)(1 - y_2), \quad z_2 = (1 - y_1)y_2,$$

and, therefore, the overall system output, interpreted as the crisp value of f at y is

$$f(y) := \frac{(1 - y_1)(1 - y_2)(1 - (1 - y_1)(1 - y_2)) + (1 - y_1)y_2(1 - y_1)y_2}{(1 - y_1)(1 - y_2) + (1 - y_1)y_2} =$$
$$y_1 + y_2 - 2y_1y_2$$

Thus our original fuzzy problem

$$\min f(x); \textit{ subject to } \{(\Re_1(x), \Re_2(x)) \mid x \in X\},$$

turns into the following crisp nonlinear mathematical programming problem

$$(y_1 + y_2 - 2y_1y_2) \to \min$$
$$\textit{subject to } \{y_1 + y_2 = 1/2, \ 0 \le y_1 < 1, \ 0 \le y_2 \le 1\}.$$

which has the optimal solution $y_1^* = y_2^* = 1/4$ and its optimal value is $f(y^*) = 3/8$.

It is clear that if there were no other constraints on the crisp values of x_1 and x_2 then the optimal solution to (6) would be $y_1^* = y_2^* = 0$ with $f(y^*) = 0$.

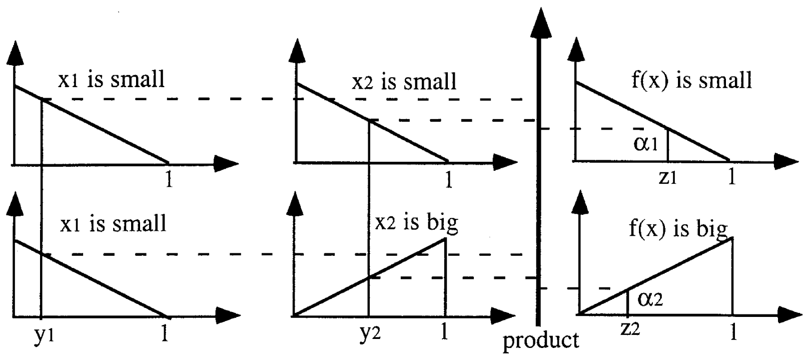

Figure 5: Takagi and Sugeno fuzzy reasoning method for Example 1.

5 Consensus reaching with interdependent utilities

In [11] we showed how the concept of interdependency can be applied to an n-party single-issue negotiation problem in which the negotiators cooperatively face a common problem or in which t he parties's interests are not diametrically opposed (variable-sum bargaining situations).
Negotiations arise from a variety of different types of disputes. Now we focuse on an n-party, single-issue negotiation in which individuals are in conflict because they want different things, but must settle for the same thing. The resolution of negotiations requires parties to reach a join decision about a settlement. Potential settlements consist of different combinations of values for the issue explicitly or implicitly under negotiation.
In a simple n-party, single-issue negotiation problem, each negotiator judges the utility of potential settlements. *Judgments of utility are usually assumed to be a function of the values of the issue (independent utilities).* Let $u_i(x)$ represent the judgment by the i-th negotiator of the utility of the potential settlement $x \in [0, 1]$. The reference points, zero and one, indicate the two possible extreme settlements of the issue.

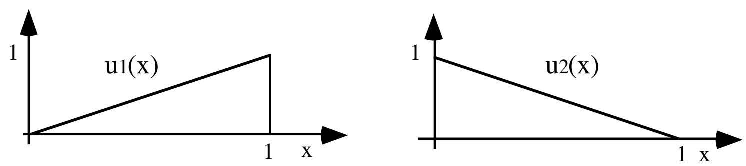

Figure 6: Linear utility functions.

To have a uniform (monoton increasing) presentation for utility functions $u_i(x)$, we introduce application functions $h_i\colon \mathbb{R} \to [0, 1]$, such that $h_i(u_i(x))$ measures the degree of satisfaction of the i-th party with the utility value $u_i(x)$ for a settlement $x \in [0,1]$, $i = 1, \ldots, n$. Usually, the application functions are of the form (the more the better)

$$h_i(t) = \begin{cases} 1 & \text{if } t \geq M_i \\ v_i(t) & \text{if } m_i \leq t \leq M_i \\ 0 & \text{if } t \leq m_i \end{cases}$$

where m_i denotes the reservation level (which represents the minimal requirement about the issue), M_i stands for the desired level on the issue and v_i is a monoton increasing function.

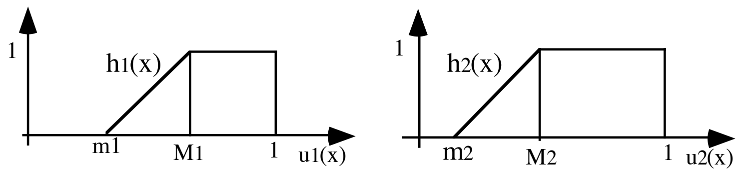

Figure 7: Linear application functions.

Negotiations consist of a ”dance” involving a sequence of proposals and counterproposals, offers and counteroffers. The uncertainty and cognitive complexity entailed in most negotiations prevent the parties from leaping to a joint agreement on their first move. Instead, negotiations tend to proceed incrementally and cautiously with the parties, attempting to ”feel their way along” to a settlement, unsure of when the level of concessions they offer meets the other's minimum reservation level, and hoping not to be taken advantage of.
The resolution of negotiations ordinarily requires settlement of differences by mutual concessions. For disputes involving multiple issues, two fundamental strategies of concession can be identified. The first consists of *compromise* - agreeing to a value intermediate between each negotiator's initial bargaining positions for each issue under dispute. The second can be described as *horsetrading* - the parties agree to trade-offs such that each obtains what he or she bargains for on certain issues, in exchange for granting the other what he or she wants on other issues.
There are two main approaches to the classification of negotiations ([34, 35]):
Distributive bargaining (DB) is commonly identified as involving the division of resources; it refers to situations in which there is a fixed supply of some resource, and one's gain is the other's loss. DB appears typically in the form of a single-issue negotiation, such as bargaining for a used car. Any reduction in the price of the car removes money from the pocket of the salesman, while any increase in the price paid removes money from the pocket of the buyer. This is also known as a *win-lose* situation. DB can also be applied to multi-issue negotiations wherein each issue is handled singularly. In DB the issues are assumed to be areas in which the parties are in conflict. As a technique, the function of DB is to resolve *pure conflicts of interest* in a *fixed-sum* negotiation.

Integrative bargaining (IB) refers to situations in which the negotiators cooperatively face a common problem or in which the parties' interests are not diametrically opposed. These constitute variable-sum bargaining situations. IB is the system of activities which is instrumental to the attainment of objectives which are not in fundamental conflict with those of the other party and which therefore can be integrated to some degree (*win-win* situation). IB may occur in a multi-issue negotiation; however, negotiators in multi-issue negotiations often bargain in a distributive manner. The IB approach attempts to steer the parties toward problem solving and away from the traditional zero-sum horse trading.
Suppose now that we have a 3-party single-issue integrative bargaining situations [11] in which we want to find a *good compromise* settlement.
Suppose that the first party is being supported by the others until it reaches the level β_{12} and β_{13}, respectively.

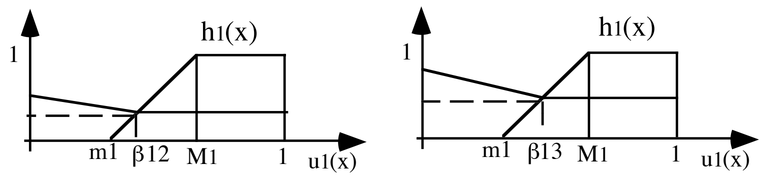

Figure 8: Feed-backs from the second and third parties.

In general, the benchmark β_{ij} denotes the level until the i-th party is being supported by the j-th party for $i, j = 1, 2, 3$. Any negotiated settlement $(\alpha_1, \alpha_2, \alpha_3)$ will satisfy the following condition (see fig. 10)

$$m_1 \le \beta_{12} \wedge \beta_{13} \le \alpha_1 \le \beta_{12} \vee \beta_{13} \le M_1,$$

$$m_2 \le \beta_{21} \wedge \beta_{23} \le \alpha_2 \le \beta_{21} \vee \beta_{23} \le M_2,$$

$$m_3 \le \beta_{32} \wedge \beta_{31} \le \alpha_3 \le \beta_{31} \vee \beta_{32} \le M_3,$$

Let us define the overall satisfaction of the group, α, with a negotiated settlement $(\alpha_1, \alpha_2, \alpha_3)$ by the minimum operator, i.e.

$$\alpha = \alpha_1 \wedge \alpha_2 \wedge \alpha_3$$

It is clear that α satisfies the following inequalities

$$\min_{i,j}\{\beta_{ij}\} \le \alpha \le \max_{i,j}\{\beta_{ij}\}$$

because from none of the parties are able to get more than $\max\{\beta_{ij}\}$, but each party can reach at least the level $\min\{\beta_{ij}\}$. If for certain i and j the benchmark level β_{ij} is given in such a way that $\beta_{ij} < m_i$ then it means that even the minimal wish of the i-th party is absolutely unacceptable for the j-th party.

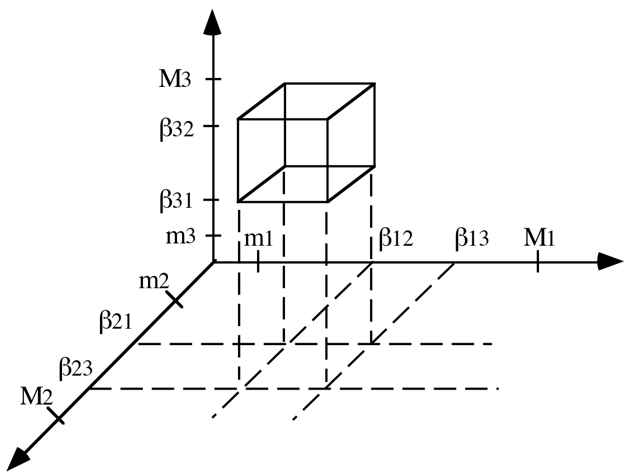

Figure 9: The efficient frontier for the negotiation problem.

If for certain i all the benchmark levels satisfy the condition

$$\beta_{ij} < m_i, \ j \neq i \tag{7}$$

then we say that the other two parties *form a coalition* againts the i-th party. However, this coalition (which is usually temporal) can result in *overall conflict* if (7) holds for all parties. In this case the negotiation dance will result in a settlement which is absolutely not acceptable by any parties.
The bencmarks β_{ij} and β_{ji} are usually determined in a preliminaries between the i-th and j-th parties before the group meeting starts.

6 Extensions

In 1997 Östermark [27] considered the interdependence concept in a dynamic setting. He showed that the approach can be naturally extended to *temporal* cases. Then he applied the temporal concept to *describe* goal conflicts in a multiperiod firm model in which the concept of static interdependence would fail. Then he generalized static membership function introduced in [6] to a dynamic membership function for both multiobjective programming and fuzzy multiobjective programming problems. In 2002 Ehrgott and Nickel [25] determined the number of objectives which are necessary to prove Pareto optimality for a given point.
In 2003 Liu, Da and Chen [28] generalized the concept of objectives interdependence introduced in [3, 6] to multidimensional problems based on the gradients of the objectives. Their novel interdependence concept reflected both the relationship and the degrees of the objectives support or conflict. Then the application functions are constructed based on the interdependence grades of the objectives, and they are aggregated by entropy regularization procedure to solve the multiobjective programming problems. The concept of interdependence has been used

by many authors, e..g. Chang, Cheng and Chen (2007) [21] when they built a fuzzy-based military officer performance appraisal system; Jain, Ramamurthy and Sundaram (2006) [22] when they examined the effectiveness of visual interactive modeling in the context of multiple-criteria group decisions; Gal and Hanne (2006) [23] when they explained the role of nonessential objectives within network approaches for MCDM; Angilella et al. (2004) [24] when assessing non-additive utility for multicriteria decision aid; Bistline et al. (1998) [26] when they introduced an interactive decision support system for solving real time scheduling problems considering customer and job priorities with schedule interruptions; Myung and Bien (2003) [29] when they designed a fuzzy multiobjective controller based on the eligibility method; Lee and Kuo [30] (1998) when they suggested a new approach to requirements trade-off analysis for complex systems; Tang and Wang (!997) [31] when they introduced an interactive approach based on a genetic algorithm for a type of quadratic programming problems with fuzzy objectives and resources,

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