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Abstract

This report examines a version of the symmetric traveling salesman problem in which the travel costs associated with arcs are given as interval ranges. The problem is optimized using robust deviation criterion. Two heuristic algorithms based on midpoint and upper point scenarios are investigated, and their efficiency on small size instances is evaluated. The quality of heuristic solutions is evaluated by means of its comparison with optimal solution obtained through mixed integer programming formulation. The efficiency of the heuristic approaches on small size instances is argued.

Keywords: robust traveling salesman problem, interval uncertainty, min-max regret and worst-case optimization, heuristics

1 Introduction

One of the most intensively investigated problems in computational combinatorial optimization and operations research is the traveling salesman problem. The problem itself sounds simple: given a set of cities and the costs of travel between each pair of them, the challenge is to find the cheapest route visiting all the cities and returning to the starting city. Though seemingly modest, the model benefits for many practical applications in e.g. genetics, telecommunications, and neuroscience. Many variants and generalizations of the problems have been studied over the years. [2]

We consider the variation of the traveling salesman problem where interval costs are not specified but given within some intervals associated with network edges. More precisely, the problem is defined on an undirected symmetric graph $G = (V, E)$ where V is a set of vertices (each vertex represents a city to be visited), and E is the set of edges of the graph (each edge represents existing connection between two given cities). An interval $[l_{ij}, u_{ij}]$, with $0 \leq l_{ij} \leq u_{ij}$ is associated with each edge $(i, j) \in E$, and it represents the possible travel times. The special interest motivated by telecommunications applications induces not to solve the interval traveling salesman problem itself, but to hedge against the worst-case realization (scenario) of problem parameters, which can be interpreted as given with uncertainty. No probability distribution is given within the intervals. Playing against worst-case scenario is generally known as robust optimization. As it was indicated in [4], in many cases the robust equivalent of a polynomially solvable problem becomes NP-hard [1]. While the classical traveling salesman problem itself is known to be NP-hard, adding robust deviation introduces additional layer into problem complexity [7].

The rest of this report is divided into three sections. Section 2 intends to use the mixed integer linear programming to calculate an optimal solution of the problem. Section 3 introduces two heuristic algorithms based on upper bound and middle point scenarios and describes the results of the computational experiments. Concluding remarks appear in the last section.

2 Robust traveling salesman problem

The basic theoretical background for the robust traveling salesman problem has been presented in [7]. A reformulation of the robust traveling salesman problem as a special mixed integer program and some preprocessing techniques were presented there.

In order to formally describe the robust deviation traveling salesman problem, we need the following definitions. Let T be a set of all Hamiltonian tours in the given graph G, and R be a set of all possible realizations of edge costs. A scenario $r \in R$ is a particular realization of the edge costs which is chosen for each edge of the graph i.e. cost $c_{ij}^r \in [l_{ij}, u_{ij}]$. The robust deviation of a tour $t \in T$ in scenario $r \in R$ is the difference between the cost of t in scenario r and the cost of the shortest tour in r:

$$dev_t^r := c_t^r - \min_{t' \in T} c_{t'}^r.$$

For a given tour $t \in T$ the worst-case scenario r_t is a scenario for which the deviation for t is maximum over all scenarios $r \in R$, i.e.

$$r_t := \arg\max_{r \in R} dev_t^r.$$

Then the difference

$$dev_t^{r_t} := c_t^{r_t} - \min_{t' \in T} c_{t'}^{r_t}$$

represents the robust deviation of t. A tour t^0 is said to be a robust Hamiltonian tour if it has the smallest robust deviation

$$t^0 := \arg\min_{t \in T} dev_t^{r_t}.$$

In other words, a tour $t^0 \in T$ is said to be a robust tour if it has the smallest (among all possible tours) maximum (among all possible scenarios) robust deviation.

Our goal in this report is to compare the results obtained by two heuristic algorithms with the optimal solution for robust version of traveling salesman problem. In order to find the optimal solution to the robust traveling salesman problem, a mixed integer linear programming has been used. The following theorem gives us the idea of how to implement the MILP formulation.

Theorem 1 [7]. *Given a tour $t \in T$, the scenario $r_t \in R$ that maximizes the robust deviation for t is the one where all the edges of tour t have the highest possible cost, and the costs of the remaining edges are at their lowest possible values, i.e. $c_{ij}^{r_t} = u_{ij}$ if $(i,j) \in t$, and $c_{ij}^{r_t} = l_{ij}$ otherwise.*

The implication of the above theorem is that, given a tour, we can compute its robustness cost by solving a classic traveling salesman problem. For each edge $(i,j) \in E$, we introduce a binary decision variable x_{ij} identifying the edges of the robust tour, i.e. $x_{ij} = 1$ if edge (i,j) is on the robust tour, and $x_{ij} = 0$ otherwise. Non-negative variable reg contains the regret term.

$$(RTSP) \qquad \min \ z(t) := \sum_{(i,j) \in E} u_{ij} x_{ij} - reg \tag{1}$$

subject to

$$reg \le \sum_{(i,j) \in E} y_{ij} l_{ij} + \sum_{(i,j) \in E} y_{ij}(u_{ij} - l_{ij}) x_{ij} \qquad \forall y \in T \tag{2}$$

$$\sum_{(j,i)\in E} x_{ji} + \sum_{(i,k)\in E} x_{ik} = 2 \qquad \forall i \in V \tag{3}$$

$$\sum_{(j,i)\in E_S} x_{ji} + \sum_{(i,k)\in E_S} x_{ik} \leq \mid S \mid -1 \qquad \forall S \subset V,\ |S| \geq 2 \tag{4}$$

$$x_{ij} \in \{0,1\} \qquad \forall\ (i,j) \in E \tag{5}$$

$$reg \in \mathbf{R}_{\geq} \tag{6}$$

Equations (1) – (6) define the robust traveling salesman problem. The crucial inequality of the MIP formulation are those in (2) which set arc costs according to the worst-case scenario $c_{ij}^{t_r}$ (as specified in Theorem 1) induced by the current tour $t \in T$ (encoded by variables y_{ij}). Constraints (3) – (4), which are standard subtour elimination constraints borrowed from the classical traveling salesman problem formulation, define the structure encoded by variables x_{ij} being a Hamiltonian tour. Constraints (3) states that exactly two edges, among those incident to a given node, must be active in a feasible solution. Inequalities (4), where E_S is the set of edges with both endpoints in S, limit the number of active edges in each possible subgraph S, in order to avoid cycles, and consequently disconnected solutions. Variable $reg \in \mathbf{R}_{\geq}$ contains information about the value of the shortest Hamiltonian tour in scenario $r_t \in R$. Thus, objective function (1) contains the difference between the cost of the tour $t \in T$ in the associated worst case scenario r_t and the cost of the shortest tour in the same scenario, i.e. it represents the robust deviation $dev_t^{r_t}$. We denote the robust path which corresponds to the optimal solution of (1) – (6) as t^0, and the corresponding optimal objective value (robust deviation) as $z(t^0)$.

3 Computational experiments

In particular, we consider two scenarios: upper point scenario $u \in R$ (all the costs at their highest possible values) and middle point scenario $m \in R$, defined as the middles of the corresponding intervals:

$$c_{ij}^m = \frac{u_{ij} + l_{ij}}{2} \quad \forall (i,j) \in E.$$

According to [3], the optimal solution of the classical traveling salesman problem on scenario m, if available, would guarantee a 2-approximation for the optimal solution of the robust traveling salesman problem itself. In the remainder of this report we will refer to the heuristic algorithm based on scenario $u \in R$ as HU, and to that based on scenario m as HM. Denote t^{HU} the Hamiltonian tour produced by HU, t^{HM} the Hamiltonian tour produced by HM, with corresponding robust deviations $dev_{t^{HU}}^{r_{t^{HU}}}$ and $dev_{t^{HM}}^{r_{t^{HM}}}$.

Type of Problem	*Average Running Time (sec.)*
$R-4-100$	0.1487
$R-4-1000$	0.1319
$R-5-100$	0.3496
$R-5-1000$	0.4008
$R-6-100$	1.7376
$R-6-1000$	2.3125
$R-7-100$	89.9329
$R-7-1000$	109.8753

Table 1: The average running time for HU and HM.

The algorithms were encoded in MATLAB R2008b. All the tests have been carried out on an Intel®core 2 Duo 2.53 GHz / 4 GB machine. The algorithm based on solving MILP formulation has two huge loops, one producing all possible tours (constraints (2)) and the other producing the subsets of V (constraints (4)). Due to computational power restrictions of the approach based on $MILP$ formulation, we restricted our analysis to small instances only. So, networks with $4, 5, 6$ and 7 vertices were analyzed. To deal with larger instances, one should use constraint propagation or branch and bound techniques to overcome the computational difficulties related to a highly constrained nature of the approach based on MILP formulation. Moreover, the analysis of HU and HM on large size instances has been previously done in [5], [6], however the information about small size instances was not presented there due to obvious targeting on large networks to test heuristics performance. Therefore, in our report we tried to shed more light on some of the white spots related to efficiency of HU and HM on small instances. For testing purposes we use a family of randomly generated instances (for detailed description see [6]). All networks represent complete graphs. The instances were generated according to the following schema: a problem of type $R-|V|-|UB|$ has $|V|$ nodes and upper bound cost u_{ij} was chosen at random from the set $\{1, 2, ..., UB\}$ while lower bound cost l_{ij} was selected again at random from the set $\{1, 2, ..., u_{ij}\}$. For each type of problems, 100 instances were produced. The average running time of HU and HM is given in Table 1.

For each problem and for each algorithm, the average relative deviation is calculated based on the following indicator (see Table 2):

$$HM\ Relative\ deviation := \frac{dev_{t^{HM}}^{s_{t^{HM}}} - z(t^0)}{z(t^0)};$$

$$HU\ Relative\ deviation := \frac{dev_{t^{HU}}^{s_{t^{HU}}} - z(t^0)}{z(t^0)}.$$

In the first and third columns of Table 2 the absolute deviations of the optimal solution and heuristic solution are given which work as a controlling tool to check the true randomness of the instances. As it can be seen there is no obvious relation between the absolute deviations. Thus we can come to this conclusion that the instances have been produced truly randomly.

In [6] it is stated that HU performs better than HM, however our experiment gives an opposite statement on small size instances. As Table 2 suggests that for the small size instances HM works much better although if we notice further it can be observed that two algorithms seem to converge to one point which is the optimal solution. While the number of vertices increases the HU algorithm gets closer to the optimality, the fact which can easily be recognized from Figure 1, where relative deviation is measured in percentages. There is one more interesting point here to note. Although in all problem types considered in the report HM has smaller relative deviation than HU, i.e. HM works more efficiently, the standard deviation indicator shows an opposite attitude when the problem gets bigger as can easily be observed from Figure 2.

Comparing the results obtained in this report with the results mentioned in [6], one can make a strong guess that it is very likely if we choose eight or nine vertices for our benchmarks, HU turns to be more efficient than HM.

There is a possible explanation for the way these two algorithms behave. Since we have chosen the number of vertices between 4 and 7, therefore the number of edges are also relatively small. If a wrong edge is chosen in HU, it can make a dramatic change in the results because we don't have sufficient number of the other edges to compensate this inappropriate choice, whereas using HM we have reduced the difference between optimal choice and the choice we have made in the algorithm.

4 Conclusions

The problem studied in this report was traveling salesman problem with interval data and robust objective to be optimized. We implemented two heuristic algorithms based on upper bound and middle point scenarios and discussed their efficiency. The computational results indicate that although these algorithms are very fast, for small size instances they are far from efficiency. However, the increase in the number of cities, improves the outcome. Based on the observations made in the previous section, we could formulate several concluding remarks:

- Since HU (as well as HM) fails to deliver a good quality solution on small instances, it may be the evidence of their weakness. The HU and HM heuristics themselves are too simple to be recognized as a reliable remedy to solve the robust traveling salesman problem.

Indeed, these approaches explore only interval structure of the problem without taking into account its topology. Thus, their usage could only be recommended for the purpose of fast generating some initial solution of reasonable quality, e.g. to generate starting solution for local search based methods. However, it should be also done with precautions, because HM and HU on large instances may produce local optima that could trap the local search mechanism preventing it from finding a global optimum.

- To make HU and HM more reliable, it would be interesting to consider some sort of hybridization between these two heuristics with some elements of randomization. Such hybridization, if implemented properly, could allow one heuristic to eliminate drawbacks of the other heuristic and vice versa, whereas randomization will make the heuristic behavior less deterministic, and therefore more flexible on "hard" instances.

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$Number$	$Type$ of $Problem$	HU $Absolute$ $deviation$	HU $Relative$ $deviation$	HU $Standard$ $deviation$	HM $Absolute$ $deviation$	HM $Relative$ $deviation$	HM $Standard$ $deviation$
1	$R-4-100$	56.1400	44.9886	10.4305	20.6500	23.1835	13.6409
2	$R-4-1000$	619.3200	49.8451	11.6740	208.6700	25.4295	13.8298
3	$R-5-100$	69.2300	40.5136	9.1765	28.5590	21.6569	12.5613
4	$R-5-1000$	587.4200	33.8842	9.1211	226.9350	16.8540	10.1522
5	$R-6-100$	40.4400	26.0877	10.2442	17.6000	13.5176	8.6724
6	$R-6-1000$	234.2300	16.9084	8.9620	113.4150	8.8952	5.9534
7	$R-7-100$	8.5600	6.4861	4.4290	6.1200	4.6617	2.7410
8	$R-7-1000$	81.5000	6.7080	4.3197	54.2600	4.6118	3.1302

Table 2: Absolute, relative and standard deviations of heuristic solutions from optimality (averaged over 100 instances).

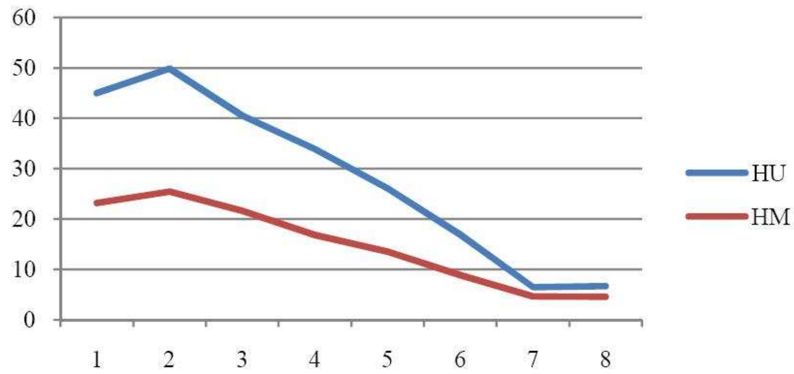

Figure 1: Relative deviation for HU and HM

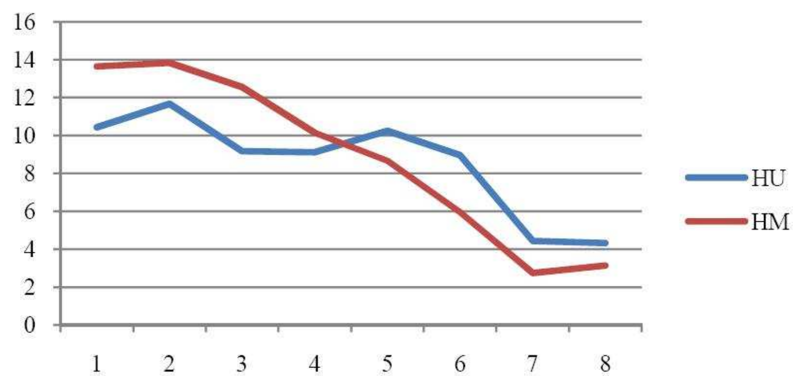

Figure 2: Standard deviation for HU and HM

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