# On MIDO Space-Time Block Codes

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***Abstract*— In this paper, the need for the construction of multiple input-double output (MIDO) space-time block codes (STBCs) is discussed, concentrating on the case of four transmitters for simplicity. Above the trivial puncturing method, i.e. switching off the extra layers in the usual multiple input-multiple output (MIMO) setting, two other, more sophisticated yet simple MIDO construction methods are proposed. The use of these general methods is then demonstrated by building explicit, sphere decodable codes using two different cyclic division algebras (CDAs). We verify by computer simulations that the newly proposed methods can compete with the puncturing method, and in some cases outperform it. Our best construction exploiting maximal orders improves even upon the punctured perfect code and the DjABBA code.**

## I. Background

Multiple-antenna wireless communication promises very high data rates, in particular when we have perfect channel state information (CSI) available at the receiver. In [1] the design criteria for such systems were developed, and further on the evolution of space-time (ST) codes took two directions: trellis codes and block codes. Our work concentrates on the latter branch.

We are interested in the coherent MIMO case where the receiver perfectly knows the channel coefficients. The received signal is

$$Y_{n_r\times\ell} = H_{n_r\times n_t}X_{n_t\times\ell} + N_{n_r\times\ell},$$

where $X$ is the transmitted codeword taken from the STBC $\mathcal{C}$, $H$ is the Rayleigh fading channel response and the elements of the noise matrix $N$ are i.i.d. complex Gaussian random variables. The indices $n_t$ and $n_r$ denote the number of transmit and receive antennas. The block length is denoted by $\ell$.

A *lattice* is a discrete finitely generated free abelian subgroup of a real or complex finite dimensional vector space called the ambient space. In the space-time setting a natural ambient space is the space $\mathcal{M}_n(\mathbf{C})$ of complex $n\times n$-matrices, i.e. $n_t = \ell = n$.

From the pairwise error probability (PEP) point of view [2], the performance of a space-time code is dependent on two parameters: *diversity gain* and *coding gain*. Diversity gain is the minimum of the rank of the difference matrix $X - X'$ taken over all distinct code matrices $X, X' \in \mathcal{C}$, also called the *rank* of the code $\mathcal{C}$. When $\mathcal{C}$ is full-rank, the coding gain is proportional to the determinant of the matrix $(X-X')(X-X')^H$, where $^H$ denotes the transpose conjugate of a matrix. The minimum of this determinant taken over all distinct code matrices is called the *minimum determinant* of the code $\mathcal{C}$. If it is bounded away from zero even in the limit as SNR $\to\infty$, the ST code is said to have the *nonvanishing determinant* (NVD) property [3]. For non-zero square matrices, being full-rank coincides with being invertible.

The *data rate* $R$ in symbols per channel use is given by

$$R = \frac{1}{n}\log_{|S|}(|\mathcal{C}|),$$

where $|S|$ and $|\mathcal{C}|$ are the sizes of the symbol set and code respectively. The *rate of a code design*, for its part, is defined as the ratio of the number of transmitted complex symbols to the decoding delay (equivalently, block length). If this ratio is equal to the delay, the code is said to have *full rate*. In literature full-rate codes are also referred to as rate-optimal.

The very first STBC for two transmit antennas was the *Alamouti code* [4] representing multiplication in the ring of quaternions. As the quaternions form a division algebra, such matrices must be invertible, i.e. the resulting STBC meets the rank criterion. Matrix representations of other division algebras have been proposed as STBCs at least in [5]-[14], and (though without explicitly saying so) [15]. The most recent work [8]-[15] has concentrated on adding multiplexing gain, i.e. MIMO applications, and/or combining it with a good minimum determinant. It has been shown in [13] that CDA-based square ST codes with the NVD property achieve the diversity-multiplexing gain (D-MG) tradeoff introduced in [16].

The codes proposed in this paper are not fully multiplexing nor do they have full rate due to the modified application requirements. This follows from the fact that the number of Rx antennas will be strictly less than the number of Tx antennas.

The paper is organized as follows. In Section II we explain why these kinds of code constructions are needed in practice and shortly discuss our solutions to the problem. The required facts from the theory of cyclic division algebras are shortly introduced in Section III. Also some examples are given there. In Section IV we depict in detail two different methods for MIDO code construction. These methods are then exemplified in Sections V and VI by two specific algebras. Finally, we provide simulation results in Section VII.

## II. Motivation and problem statement

In some applications it is well possible that the number of Rx antennas is required to be strictly less than the number of Tx antennas. A typical example is a cellular phone downlink with two receivers exploiting polarization. Due to the limited size of 3+G mobile phones and DVB-H (Digital Video Broadcasting-Handhelds) user equipment, only a very small number of antennas fits at the end user site. For this kind of an application, the minimum delay MIMO constructions arising from the theory of cyclic division algebras have to be modified. For simplicity, we will concentrate on the 4Tx+2Rx antennas MIDO case. If we could afford four Rx antennas, the task would be easy – just to use the $4 \times 4$ minimum delay, rate-optimal CDA based construction transmitting 16 Gaussian numbers (= complex integers) in four time slots, i.e. four in each time slot. Now, however, the reduced number of Rx antennas limits the transmission down to two Gaussian numbers per each time slot.

We have come up with two different types of solutions to this problem. Both solutions take advantage of cyclic division algebras and yield rate two codes with a nonvanishing determinant. One idea is to first pick an index two division algebra with a center that is four-dimensional over $\mathbf{Q}$, form an isomorphic copy of it and then use them as blocks in a $4\times 4$ code matrix. Another possibility is to take the usual $4\times 4$ MIMO code, but choose the elements in the matrix from an intermediate field instead of the maximal subfield. The exact principles for the constructions are given in Sections IV-A and IV-B.

## III. Cyclic division algebras

The theory of cyclic algebras and their representations as matrices are thoroughly considered in [6] and [17]. We are only going to recapitulate the essential facts here.

In the following, we consider number field extensions $E/F$, where $F$ denotes the base field and $F^*$ (resp. $E^*$) denotes the set of the non-zero elements of $F$ (resp. $E$). The rings of algebraic integers are denoted by $\mathcal{O}_F$ and $\mathcal{O}_E$ respectively. Let $E/F$ be a cyclic field extension of degree $n$ with Galois group (= the set of automorphisms of $E$ such that $F$ is fixed under them)

$$\mathrm{Gal}(E/F) = \langle\sigma\rangle = \{id = \sigma^n, \sigma, \sigma^2, ..., \sigma^{n-1}\},$$

where $\sigma$ is the generator of the cyclic group. Let

$$\mathcal{A} = (E/F, \sigma, \gamma)$$

be the corresponding cyclic algebra of degree $n$ ($n$ is also called the *index* of $\mathcal{A}$ and in practice it determines the number of transmitters), that is

$$\mathcal{A} = E \oplus uE \oplus u^2E \oplus \cdots \oplus u^{n-1}E,$$

with a noncommuting element $u \in \mathcal{A}$ such that $eu = u\sigma(e)$ for all $e \in E$ and $u^n = \gamma \in F^*$. An element

$$x = x_0 + ux_1 + \cdots + u^{n-1}x_{n-1} \in \mathcal{A}$$

has the following representation as a matrix $A =$

$$\begin{pmatrix} x_0 & \gamma\sigma(x_{n-1}) & \gamma\sigma^2(x_{n-2}) & \cdots & \gamma\sigma^{n-1}(x_1) \\ x_1 & \sigma(x_0) & \gamma\sigma^2(x_{n-1}) & & \gamma\sigma^{n-1}(x_2) \\ x_2 & \sigma(x_1) & \sigma^2(x_0) & & \gamma\sigma^{n-1}(x_3) \\ \vdots & & & & \vdots \\ x_{n-1} & \sigma(x_{n-2}) & \sigma^2(x_{n-3}) & \cdots & \sigma^{n-1}(x_0) \end{pmatrix}. \tag{1}$$

*Definition 3.1:* The determinant (resp. trace) of the matrix $A$ is called the *reduced norm* (resp. *reduced trace*) of the element $x \in \mathcal{A}$ and is denoted by $nr(x)$ (resp. $tr(x)$).

*Definition 3.2:* An algebra $\mathcal{A}$ is called *simple* if it has no nontrivial ideals. An $F$-algebra $\mathcal{A}$ is *central* if its center $Z(A) = \{x \in \mathcal{A} \mid xx' = x'x \text{ for all } x' \in \mathcal{A}\} = F$.

All algebras considered here are finite dimensional associative central simple algebras over a field. From now on, we identify the element $x$ of an algebra with its standard matrix representation defined above in (1).

The next proposition describes a ring from where the elements are drawn in a typical CDA based MIMO space-time block code.

*Proposition 3.1:* Let us define the ring

$$\Lambda_{\mathcal{A}} = \{x_0 + \cdots + u^{n-1}x_{n-1} \mid x_i \in \mathcal{O}_E\}$$

$\subseteq \mathcal{A} = (E/F, \sigma, \gamma)$. For any non-zero element $x \in \Lambda_{\mathcal{A}}$ its reduced norm $nr(x)$ is a non-zero element of the ring of integers $\mathcal{O}_F$ of the center $F$. In particular, if $F$ is an imaginary quadratic number field, then the minimum determinant of the lattice $\Lambda_{\mathcal{A}}$ is nonvanishing and equal to one.

*Proof:* See [18, Theorem 10.1, p. 125]. ∎

The next proposition provides us with a condition when an algebra is a division algebra, i.e. each of its non-zero elements has a multiplicative inverse. This is an old result of A. A. Albert [19].

*Proposition 3.2:* The algebra $\mathcal{A} = (E/F, \sigma, \gamma)$ of degree $n$ is a division algebra if and only if the smallest factor $t \in \mathbf{Z}_+$ of $n$ such that $\gamma^t$ is the norm of some element in $E^*$ is $n$.

*Proof:* See [19, Theorem 11.12, p. 184]. ∎

For instance, let us define two cyclic division algebras that will be later on used in the actual code constructions. We denote by $\zeta_n = e^{2\pi i/n}$ the primitive $n$th root of unity and the imaginary unit by $i = \sqrt{-1}$.

### A. Perfect algebra

In [9] the authors presented the so-called perfect codes that satisfy certain, quite strict, design criteria and hence perform very well in computer simulations. The underlying algebra in their $4 \times 4$ construction is the cyclic division algebra

$$\begin{aligned} \mathcal{PA} &= (E/F, \tau, \gamma) \\ &= \{x = x_0 + ux_1 + u^2x_2 + u^3x_3 \mid x_i \in E\} \end{aligned}$$

with $E/F = \mathbf{Q}(\theta, i)/\mathbf{Q}(i)$, $\gamma = i$, $\theta = \zeta_{15} + \zeta_{15}^{-1} = 2\cos(2\pi/15)$, and $\tau(\theta) = \theta^2 - 2$. The corresponding perfect code is

$$PC = \{ax \mid x \in \Lambda_{\mathcal{PA}} \text{ (cf. Prop. 3.1)},\ a = 1 - 3i + i\theta^2\},$$

where $\mathcal{I} = \langle a \rangle$ is an ideal of $\mathcal{O}_E$.

Moreover, a change of basis given by

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & 0 & 1 \\ -1 & -3 & 1 & 1 \end{pmatrix}$$

is required for obtaining an orthogonal basis.

### B. Cyclotomic algebra

The MIDO codes obtained from the perfect algebra will be compared with the algebra

$$\begin{aligned} \mathcal{CA} &= (E/F, \tau, \gamma) \\ &= \{x = x_0 + ux_1 + u^2x_2 + u^3x_3 \mid x_i \in E\} \end{aligned}$$

with $E/F = \mathbf{Q}(\xi = \zeta_{16})/\mathbf{Q}(i),\ \gamma = 2 + i$, and $\tau(\xi) = i\xi$.

This algebra has appeared also in at least [10] and [14].

## IV. Constructing MIDO lattices

A straightforward way to obtain MIDO lattices would be just to 'switch off the extra layers' in a MIMO setting, i.e. by puncturing. In the case of 4Tx+2Rx antennas this would mean that in (1) we set e.g. $x_1 = x_3 = 0$ in order to transmit a limited number of 8 Gaussian numbers that can be received with only two receivers. In what follows we present two more sophisticated methods for constructing MIDO lattices.

### A. Construction by Method I

*Proposition 4.1:* Let $\mathcal{A} = (E/F, \tau, \gamma) = E + uE + u^2E + u^3E$ be an arbitrary index four division algebra. Then we also have an index two division algebra $\mathcal{B} = (E/L, \sigma, \gamma) = E + u^2E$, where the center $L$ is fixed by $\sigma = \tau^2$. The Galois groups are $\mathrm{Gal}(E/F) = \langle \tau \rangle$, $\mathrm{Gal}(E/L) = \langle \sigma = \tau^2 \rangle$, and $\mathrm{Gal}(L/F) = \langle \tau_{|L} \rangle$.

*Proof:* For the extension field degrees we clearly have $[E : L] = [L : F] = 2$. Let $E = L(\alpha),\ L = F(\beta)$ for some primitive elements $\alpha \in E,\ \beta \in L$, and $\mathrm{Gal}(L/F) = \langle \nu \rangle$. Then $\nu^2(\beta) = \beta$ and the automorphism $\nu$ can be extended to an automorphism $\rho:\ E \to E$ by defining $\rho(\alpha) = \tau(\alpha)$ and $\rho_{|L} = \nu$. The field $L$ is the center of the algebra $\mathcal{B}$ as it is fixed by $\sigma$. It remains to prove that for all $x \in E^*,\ \gamma \neq N_L^E(x)$. Let us make a counter assumption that for some $x \in E^*, N_L^E(x) = \gamma$. As $\gamma \in F^*$, $\tau(\gamma) = \gamma$. Whence it follows that $N_F^E(x) = N_F^L(N_L^E(x)) = N_F^L(\gamma) = \gamma\tau(\gamma) = \gamma^2$. This is a contradiction as the algebra $\mathcal{A}$ is a division algebra. ■

An element $b = x_0 + ux_1,\ x_0, x_1 \in E$ of the algebra $\mathcal{B}$ has a representation as a $2 \times 2$ matrix

$$B = \begin{pmatrix} x_0 & \gamma\sigma(x_1) \\ x_1 & \sigma(x_0) \end{pmatrix}. \tag{2}$$

However, we can afford a $4 \times 4$ packing as we are using four transmitters. This can be achieved by using the isomorphism $\tau$. Let us denote by $\tau(\mathcal{B}) = (E/F, \sigma, \tau(\gamma))$ the isomorphic copy of $\mathcal{B}$ and the respective matrix representation by

$$\tau(B) = \begin{pmatrix} \tau(x_0) & \tau(\gamma)\tau(\sigma(x_1)) \\ \tau(x_1) & \tau(\sigma(x_0)) \end{pmatrix}. \tag{3}$$

*Proposition 4.2:* Let $\mathcal{O}_E$ be the ring of algebraic integers of $E$ and $F = \mathbf{Q}(i)$. The lattice

$$\mathcal{C}_1 = \left\{ M = M(x_0, x_1) = \begin{pmatrix} B & 0_{2\times 2} \\ 0_{2\times 2} & \tau(B) \end{pmatrix} \;\middle|\; x_0, x_1 \in \mathcal{O}_E \right\}$$

built from (2) and (3) has a nonvanishing determinant $\det(\mathcal{C}_1) \in \mathbf{Z}[i]$. Thus, the minimum determinant is equal to one.

*Proof:* According to Definition 3.1 and Proposition 3.1,

$$\begin{aligned} \det(M) &= \det(B)\det(\tau(B)) = nr(B)nr(\tau(B)) \\ &= nr(B)\tau(nr(B))) = N_{L/F}(nr(B)) \in \mathbf{Z}[i], \end{aligned}$$

and hence $|\det(M)| \geq 1$. ■

### B. Construction by Method II

*Proposition 4.3:* Let $\mathcal{A}$ be as in Proposition 4.1. If in the matrix $(1)$ the elements $x_i$ are restricted to belong to $L$ (rather than to $E$), we obtain a division algebra $\mathcal{A}'$ with the center $F[u^2]$.

*Proof:* Obviously also the algebra $\mathcal{A}'$ is a division algebra as it is contained in $\mathcal{A}$. As in Proposition 4.1, $L$ is fixed by $\sigma = \tau^2$, and therefore $lu^2 = u\tau(l)u = u^2\tau^2(l) = u^2\sigma(l) = u^2l$ for all $l \in L$. The center $F$ of $\mathcal{A}$ is thus extended by the element $u^2$. ■

*Proposition 4.4:* Let $\mathcal{O}_L$ be the ring of algebraic integers of $L$ and $F = \mathbf{Q}(i)$. The lattice

$$\mathcal{C}_2 = \left\{ \begin{pmatrix} x_0 & \gamma\tau(x_3) & \gamma x_2 & \gamma\tau(x_1) \\ x_1 & \tau(x_0) & \gamma x_3 & \gamma\tau(x_2) \\ x_2 & \tau(x_1) & x_0 & \gamma\tau(x_3) \\ x_3 & \tau(x_2) & x_1 & \tau(x_0) \end{pmatrix} \;\middle|\; x_i \in \mathcal{O}_L \right\}$$

has a nonvanishing determinant $\det(\mathcal{C}_2) \in \mathbf{Z}[i]$. Thus, the minimum determinant is equal to one.

*Proof:* This immediately follows from the way of construction. ■

## V. MIDO codes using $\mathcal{PA}$

For $\mathcal{PA}$ (cf. Section III-A) we have the nested sequence of fields $F \subseteq L \subseteq E$ with $L = \mathbf{Q}(i, \sqrt{5})$. As $\tau(\sqrt{5}) = -\sqrt{5}$, the field $L$ is fixed by $\sigma = \tau^2$ as indicated in Proposition 4.1.

### A. Method I

By embedding the algebra $(E/L, \sigma, i)$ as in Proposition 4.2 we obtain the MIDO code

$$\mathcal{PA}_1 \subseteq \left\{ \begin{pmatrix} x_0 & i\sigma(x_1) & 0 & 0 \\ x_1 & \sigma(x_0) & 0 & 0 \\ 0 & 0 & \tau(x_0) & i\tau(\sigma(x_1)) \\ 0 & 0 & \tau(x_1) & \tau(\sigma(x_0)) \end{pmatrix} \;\middle|\; x_i \in \mathcal{O}_E \right\}.$$

As the center is now $L$ with $[L : \mathbf{Q}(i)] = 2$ and $\mathcal{O}_L = \mathbf{Z}[i, \mu = (1+\sqrt{5})/2]$, the elements $x_i$ in the matrix are of the form $a_1 + a_2\mu + a_3\theta + a_4\mu\theta$, where $a_i \in \mathbf{Z}[i]$ for all $i$. Hence, the code rate is $8/4 = 2$.

### B. Method II

Let us now use the method from Section IV-B. We get the MIDO code

$$\mathcal{PA}_2 \subseteq \left\{ \begin{pmatrix} x_0 & i\tau(x_3) & ix_2 & i\tau(x_1) \\ x_1 & \tau(x_0) & ix_3 & i\tau(x_2) \\ x_2 & \tau(x_1) & x_0 & i\tau(x_3) \\ x_3 & \tau(x_2) & x_1 & \tau(x_0) \end{pmatrix} \middle| \, x_i \in \mathcal{O}_L \right\}.$$

Each of the elements $x_i$ is of the form $a_1 + a_2\mu$, where $a_1, a_2 \in \mathbf{Z}[i]$. Thus, the code rate is again equal to two.

## VI. MIDO codes using $\mathcal{CA}$

The algebra $\mathcal{CA}$ (cf. Section III-B), for its part, has the nested sequence of fields $F \subseteq L \subseteq E$ with $L = \mathbf{Q}(s = \zeta_8)$. As $\tau(s) = -s$, the field $L$ is fixed by $\sigma = \tau^2$.

### A. Method I

Again by embedding the algebra $(E/L, \sigma, 1 + s - i)$ as in Proposition 4.2, the MIDO code $\mathcal{CA}_1 \subseteq$

$$\left\{ \begin{pmatrix} x_0 & (1+s-i)\sigma(x_1) & 0 & 0 \\ x_1 & \sigma(x_0) & 0 & 0 \\ 0 & 0 & \tau(x_0) & (1-s-i)\tau(\sigma(x_1)) \\ 0 & 0 & \tau(x_1) & \tau(\sigma(x_0)) \end{pmatrix} \right\}$$

with $x_i \in \mathcal{O}_E$ is obtained. Note that we have chosen the nonnorm element $\gamma = 1+s-i\mathcal{O}_L \backslash \mathcal{O}_F$ with a smaller absolute value in order to get some energy savings.

The center is $L$ with $[L : \mathbf{Q}(i)] = 2$ and $\mathcal{O}_L = \mathbf{Z}[s]$. The elements $x_i$ in the matrix are of the form $a_1 + a_2 s + a_3\xi + a_4 s\xi$, where $a_i \in \mathbf{Z}[i]$ for all $i$, and so the code rate is 2.

### B. Method II

Let us then construct a MIDO code with the method from Section IV-B. This time we have

$$\mathcal{CA}_2 \subseteq \left\{ \begin{pmatrix} x_0 & \gamma\tau(x_3) & \gamma x_2 & \gamma\tau(x_1) \\ x_1 & \tau(x_0) & \gamma x_3 & \gamma\tau(x_2) \\ x_2 & \tau(x_1) & x_0 & \gamma\tau(x_3) \\ x_3 & \tau(x_2) & x_1 & \tau(x_0) \end{pmatrix} \middle| \, x_i \in \mathcal{O}_L \right\}$$

with $\gamma = 1 + s - i$.

Each of the elements $x_i$ is of the form $a_1 + a_2 s$, where $a_1, a_2 \in \mathbf{Z}[i]$. Thus, the code rate equals two.

## VII. Simulation results

In Figure 1, the different construction methods are denoted by: $0 =$ Puncturing method, $1 =$ Method 1, and $2 =$ Method 2.

First of all, we have to admit that we have not carried out optimization as much as would have been possible. For example, the use of ideals has not been taken advantage of, except in the case of the punctured ($x_1 = x_3 = 0$, cf. (1)) Perfect code $\mathcal{PA}_0$ and the code $\mathcal{PA}_1$, for which we used the ideal given in III-A. The codes $\mathcal{PA}_2$ and $\mathcal{CA}_2$ used for the simulations in Figure 1 are exactly as given in Sections V-B and VI-B.

The codes $\mathcal{CA}_1$, $\mathcal{PA}_2$, $\mathcal{PA}_1$, and $\mathcal{PA}_0$ perform more or less equally. The code $\mathcal{CA}_2$ loses to these by 0.2-0.7 dB, depending on the SNR. Next comes $\mathcal{CA}_0$ ($x_1 = x_3 = 0$), losing still by $0.7 - 1$ dB to $\mathcal{CA}_2$. A sphere decoder was used for decoding the lattices.

The best code is $\mathcal{CA}_1\ MAX$ gotten by combining Method 1 with the use of a maximal order [22] within the algebra $\mathcal{CA}$. It outperforms the next best code by approximately $0.5 - 1$ dB. In [23] the authors show that the DjABBA code wins the punctured Perfect code by $0.5$ dB or less at the rate 4 bpcu. Hence, our code improves even upon the DjABBA code - or at least ties with it.

It seems that the best construction method depends on the very algebra that is in use. Figure 1 shows that the puncturing method is not always the first choice, hence proving the point of new construction methods. We also want the reader to note that in principle, MIDO codes can be designed just by using the standard CDA based MIMO code with a smaller constellation. Nevertheless, this destroys the lattice structure and causes exponential complexity at the receiver.

## VIII. Conclusions and further study

Two nontrivial methods for constructing MIDO lattices were proposed and illustrated by two explicit example algebras. As compared to the puncturing method of just switching off the extra layers in a MIMO code, these new methods perform very well when we note that we have not yet optimized them at all as opposed to the heavy optimization carried out for the perfect code. E.g. the codes can be pre- and postmultiplied by any complex matrix of determinant one without affecting neither its density nor its good minimum determinant.

An open question is how to better parameterize these MIDO constructions. It would also be interesting to find out which combinations of Tx and Rx antennas with #Tx $>$ #Rx are possible and efficient. E.g. the case of 6Tx+3Rx is interesting.

At this point, we cannot say whether one of the methods is universally better than the others. The simulations we have carried out this far indicate that the optimality depends on the algebra in question. We are hoping to get deeper insight to this as we do some further optimization and parameterizing, and analyze several different algebras. The preliminary results however tell us that sometimes we are better off not using the puncturing method.

In [20], [21], and [22] we have studied the use of maximal orders [18] in the design of dense CDA based MIMO STBCs. The same ideas can be adapted to the MIDO scheme as well. We expect the constructions arising from maximal orders to perform better than the ones within natural orders due to the increased density. The preliminary results shown here are indeed promising as we managed to beat or tie with the punctured Perfect code and the DjABBA code. It would be worthwhile to see if a maximal order of the Perfect algebra would perform even better than the code $\mathcal{CA}_1\ MAX$. But this is for further study.

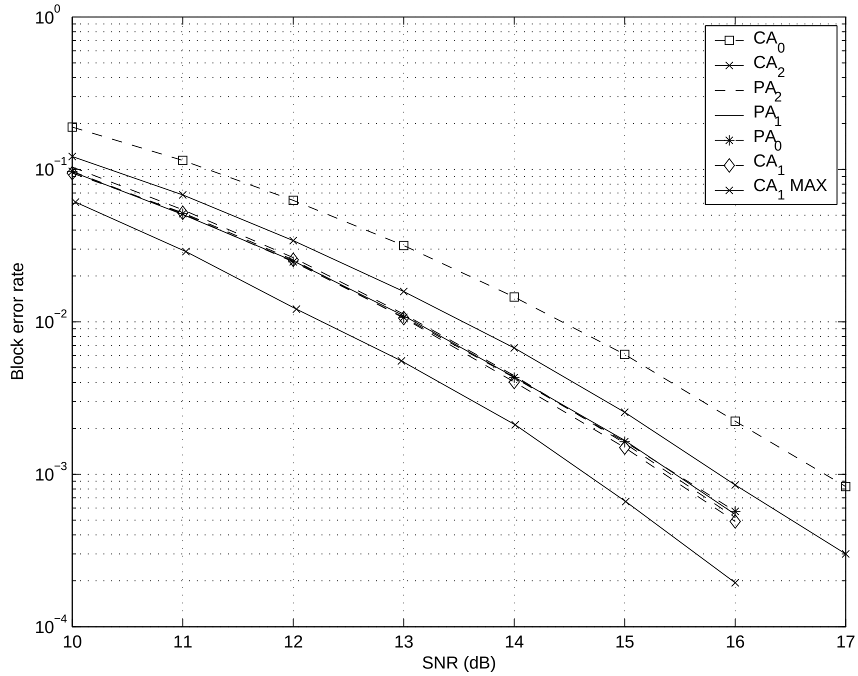


Fig. 1. MIDO block error rates at 4 bpcu with #Tx=4, #Rx=2.

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