OVERSATURATING SYNCHRONOUS CDMA SYSTEMS USING COLLABORATIVE CODING

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Abstract - Oversaturated synchronous CDMA system is proposed based on collaborative coding, where data bits of group of $L + s$ users are jointly one-to-one mapped onto 2^{L+s} L-dimensional signal vectors. Instead of unique signature per each user, L-dimensional signature subspace is used to transmit bits of $L + s$ users ($s > 0$). All signature subspaces are orthogonal to each other which simplifies optimal receiver structure. With signal space dimension N, the number of users is $K = N(1 + s/L)$. Preferable collaborative codes are found using the sphere packing theory for $L = 2 \ldots 5$, $s = 1, 2$. Trade-off between oversaturation efficiency and energy loss/gain is evaluated against conventional non-oversaturated orthogonal CDMA.

Keywords - CDMA, oversaturation, collaborative coding, signal design

I. INTRODUCTION

In conventional direct sequence code division multiple access (DS-CDMA) each user is assigned unique spreading code or signature [1,2]. In synchronous CDMA (S-CDMA), where all signatures are strictly time synchronized at the receiver input, the best choice for signatures is orthogonal set, provided that the number of users K does not exceed signal space dimension N. In such case multiple access interference (MAI) is totally eliminated. For example, orthogonal signatures (Walsh functions) are used in the downlink of IS-95 and UMTS mobile radio standards [2,3]. Even in the uplink of UMTS, orthogonal signatures are involved to realize multicode channelization, i.e. parallel transmission of data of one user through a number of dedicated physical traffic channels to increase overall data rate [3].

The modern wireless telecommunications research strives for a higher capacity, i.e. number of users served. Within limited spectral resource and predefined data rate, signal space dimension is constrained. Thus, further increase of K leads to *oversaturated* CDMA ($K > N$). In these circumstances MAI becomes unavoidable and the performance of conventional single-user receiver is poor. At the same time optimal (multiuser) receiver [4] may appear very complex unless special requirements are imposed on oversaturated signature ensemble design.

This article inspects methods to improve capacity of S-CDMA when compared to conventional orthogonal signaling with as simple as possible receiver structure. First, oversaturation strategy based on *signature per user* idea is briefly reviewed. Then, the idea of individual signatures is abandoned and oversaturation based on *collaborative coding* is presented. The signal design is accomplished with the aid of the sphere packing theory. Minimum distances for several optimal and near optimal signal configurations are found and energy efficiency versus oversaturation efficiency is evaluated against conventional orthogonal signaling. It is seen, that collaborative coding possesses significantly higher performance than signature per user oversaturation.

II. OVERSATURATION ON THE SIGNATURE PER USER BASIS

Numerous proposals how to arrange oversaturated CDMA, provided unique signature is attached to every user, have been published. Originally in [5] a method of accommodating $K = N + M$ users in N-dimensional signal space that does not compromise the minimum Euclidean distance of orthogonal signaling was presented. High energy efficiency of this scheme is obtained at the price of relatively low oversaturation efficiency $e_{ov} = K/N = 1 + M/N \approx 1.33$, and rather complicated multiuser receiver. These problems were further addressed in [6-11]. A fast receiving algorithm for optimal multiuser detection in the situation where users' signals have tree-like correlation coefficient structure was proposed in [6]. Another kind of receiver simplification is presented in [7], where signals are divided into groups that are orthogonal to each other. Similar idea is exploited in [8], where N-dimensional global signal space is divided to N/L L-dimensional orthogonal subspaces. Each subspace is allocated to $L+s$ users, which results in oversaturation efficiency $e_{ov} = 1 + s/L$. There are 2^{L+s} (number of different binary linear combinations of $L + s$ signatures) possible resulting group signals in any L-dimensional subspace due to multiplication of any signature by a user's antipodal data bit. Constellation of these signals ($(L+s, L)$ constellation) is designed to maximize the minimum distance between all possible pairs of group signals. In the receiving end, $(L+s)$-user multiuser receiver is employed. The receiver is simple due to small values of L and s.

III. OVERSATURATION BASED ON COLLABORATIVE CODING

Rigid one-to-one correspondence between signatures and users may be considered as needlessly binding in some cases. For instance, in the cellular radio downlink, the data of all users is under control of the base station and may be in principle encoded jointly on the basis of so called collaborative coding or collaborative coding multiple access (CCMA) [12]. The oversaturation scheme proposed here is a particular version of CDMA/CCMA. In general, the idea of considered oversaturation CDMA scheme was formulated by Fan and Darnell in [13]. In our interpretation it implies dividing N-dimensional space into N/L orthogonal subspaces each of dimension L. Every subspace is used to transmit data of $L+s$ users as described in [8], but with no assignment of specific signatures to users. Instead, we have a set of N orthogonal signatures divided into N/L subsets, each subset being allocated to a specific group of $L+s$ users whose 2^{L+s} different bit patterns are one-to-one mapped onto 2^{L+s} L-dimensional signal vectors.

Now, we are not bound by the restraint of [8] that group signals are only linear combinations of $L+s$ bit-manipulated fixed L-dimensional vectors. It allows us to find the best possible $(L+s, L)$ constellation of subspace signal vectors with globally maximal distinguishability between them, which utilizes available time-frequency resource in a most effective way.

Receiver intended to retrieve i-th data stream is actually tuned to the signature subset covering i-th user. It is not subject to MAI from the other received signature subsets because all of these subsets are orthogonal. Receiver first restores bit pattern of all $L+s$ users knowing the rule of correspondence between transmitted signals and users' bit patterns, and afterwards abandons needles data of all users but the i-th one.

Now, the key issue is discussed: How to choose an appropriate $(L+s, L)$ constellation or collaborative code?

IV. SIGNAL DESIGN

As a reference for further comparison, table 1 gives the summary of results of [8]: maximized minimum squared distance $d^2_{\min}$ between group signals in L-dimensional subspace when $L+s$ signatures are allocated to $L+s$ users versus oversaturation efficiency. The rightmost column of the table presents energy loss γ in dB due to oversaturation against orthogonal signaling (for the latter $s=0$, $d^2_{\min}=4$).

Table 1: Summary of results of [8]

L $(s=1)$	e_{ov}	$d^2_{\min}$	γ [dB]
2	1.50	$2(3-\sqrt{5})$	4.19
3	1.33	$2(4-\sqrt{7})$	1.69
4	1.25	4	0

In our formulation, average energy E over all 2^{L+s} different signal vectors is set equal to the total energy of signatures employed in the scheme of [8]:

$$E = \frac{1}{2^{L+s}} \sum_{k=1}^{2^{L+s}} E_{ck} = (L+s)E_b, \tag{1}$$

where E_{ck} is the energy of k-th signal vector in the collaborative coding scheme and E_b is energy per bit per signature in the signature per user scheme. Such a normalization puts both schemes into equivalent conditions considering energy consumption.

From the geometrical point of view, finding constellation of maximally distant 2^{L+s} L-dimensional vectors of fixed average energy (i.e. squared length) may be treated in terms of a densest packing of 2^{L+s} spheres in the L-dimensional space. Classical sphere packing theory [14] aims to find such a packing of equal nonoverlapping spheres that the ratio between sum of volumes of all the packed spheres and the volume of the obtained packing is maximal. Globally optimal results are not known for space dimension $L \geq 3$. However, among lattice structures optimal packing is known if space dimension $L \leq 8$. Lattice packing is defined by the following property: origin is one of sphere centers, and if there are sphere centers $\mathbf{u}$ and $\mathbf{v}$, then there are also spheres with centers $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$, i.e. possible center points form an additive group [14].

It should be noted, that in our case, density criterion according to (1), differs from a classical approach, since spheres should be packed to guarantee minimal average squared distance of sphere centers from the origin. However, as it will be shown, the solutions taken from classical sphere packing theory may assist (at least for small dimensions L) in finding good collaborative codes. Specific examples starting with oversaturation of L-dimensional space by only one extra user ($s=1$) are presented next.

A. *Two-dimensional subspace* ($e_{ov}=1.50$)

In the case of $(3,2)$ constellation spheres reduce to circles. Thus, the aim is to place $2^{L+1}=8$ circles on the plane with as great as possible equal diameter at average squared distance from the origin fixed by (1). Normalizing E_b to unity for convenience, the normalized $d^2_{\min}$ is then calculated for any circle packing from (1). First, consider the densest 2D lattice packing [14] of fig. 1. Directly from the fig. 1 and (1)

$$E = \frac{1}{8}\left(1\cdot 0 + 6\cdot d^2_{\min} + 1\cdot 3d^2_{\min}\right) = 3$$
$$\Rightarrow d^2_{\min} = \frac{8E}{9} = \frac{8}{3} \approx 2.67.$$

Comparison of this figure with minimum distance of orthogonal signaling mentioned earlier ($d^2_{\min}=4$) shows energy loss due to oversaturation $\gamma = 4/(8/3) = 3/2 \approx 1.76$ dB, which is significantly (2.4 dB) lower than in optimal signature per user $(3,2)$ constellation (see table 1). Although, this packing

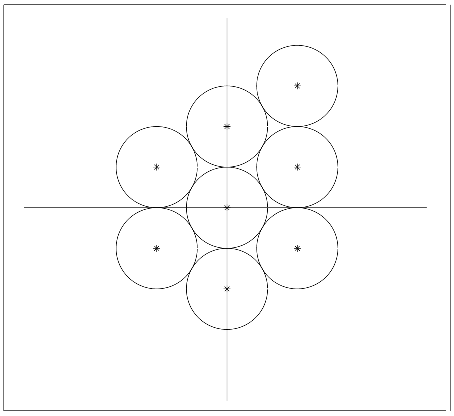

Figure 1: Signal constellation after the densest packing

is the densest, the minimum distance can still be increased. The asymmetry of the constellation allows to shift it so that its centroid falls into the origin. Such a displacement (see fig. 2) lowers loss to $\gamma \approx 1.58$ dB.

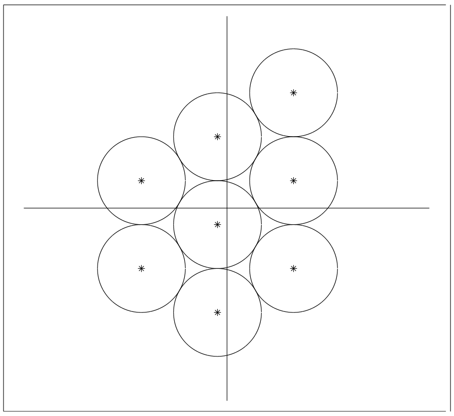

Figure 2: Signal constellation after balancing the densest packing

The asymmetric constellation can be problematic from the receiver implementation point of view. Let us press the isolated signal point (circle) to the ring surrounding the circle in the origin. The resulting constellation is shown in fig. 3. The minimum squared distance in this case $d^2_{\text{min}} \approx 2.58$, which in comparison to orthogonal signaling results in loss of $\gamma \approx 1.90$ dB. When compared to best possible packing, this suboptimal constellation is only 0.32 dB worse.

If for some reason, the zero vector cannot be the one of signals in constellation, the best constellation when distance property is considered, is illustrated in fig. 4, which provides $d^2_{\text{min}} \approx 2.54$ and the $\gamma \approx 1.98$ dB.

When equal energy signals are preferred, signal constellation becomes familiar 8-PSK pattern, where $d^2_{\text{min}} \approx 1.76$ and $\gamma \approx 3.57$ dB. This loss is rather big, but it is still 0.62 dB better than the value obtained with the optimal 2D-signal set in table 1. The constellation is shown in fig. 5.

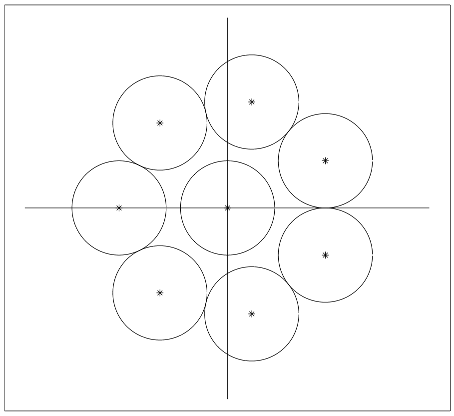

Figure 3: Signal constellation with zero vector, other points have equal energy

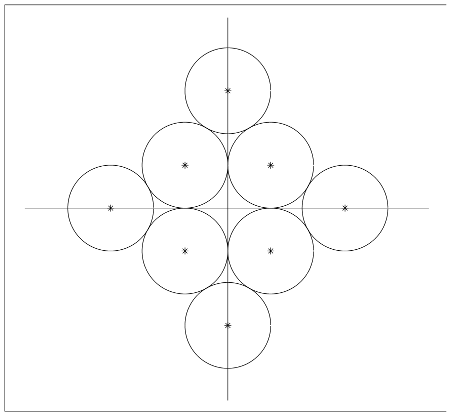

Figure 4: Symmetrical signal constellation without zero-vector

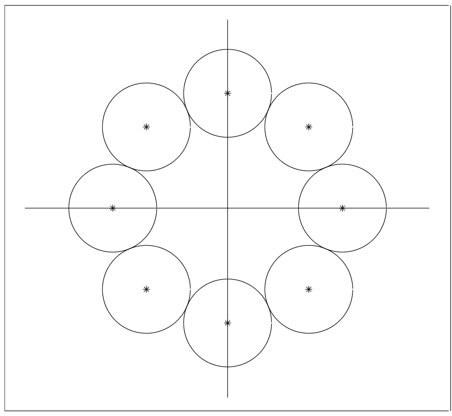

Figure 5: Equal energy constellation

B. *Three-dimensional subspace* ($e_{ov} = 1.33$)

The densest lattice packing in 3D-space is well known face-centered cubic (fcc) lattice [14]. Centers for spheres are obtained by taking points of cubic lattice whose coordinates add up to an even integers [14]. Thus, the signal matrix after adjusting the average energy to $E = 4E_b$ can be, for example,

$$\mathbf{S} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & -1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \\ -2 & 0 & 0 \end{pmatrix} \sqrt{\frac{4E}{9}}. \tag{2}$$

Squared minimum distance for this $(4, 3)$ constellation is $d^2_{\text{min}} \approx 3.56$, which implies that the loss against orthogonal signals is $\gamma \approx 0.51$ dB. Again, this constellation is asymmetric. If the centroid of constellation is balanced to the origin the loss can be reduced to 0.42 dB.

The symmetric version of fcc without the signal point in origin, $(0\,0\,0)$ being replaced for example by $(0\ -2\ 0)$, results in loss $\gamma \approx 0.97$ dB.

If equal energy constellation is of interest, center coordinates can be taken from [15], where points on a sphere surface having maximal separation are found. This choice leads to loss $\gamma \approx 1.10$ dB.

If zero vector is allowed to be a signal point and other signal points have equal distance from the origin, the packing provides $\gamma \approx 0.61$ dB.

Signal optimisation results for both two- and three-dimensional cases are summarized in table 2, where the following notation is used to specify constellations universally. It also fits for cases $L = 4, 5$, which are analyzed in the following sections:

1: Packing on sphere surface - all spheres have equal distance from the origin (fig. 5)

2: Volume packing - symmetric constellation, no signal point in the origin (fig. 4)

3: Packing on sphere surface - one signal point in the origin, others have equal distance from the origin (fig. 3)

4: Volume packing - the densest lattice (fig. 1)

5: Volume packing - the densest lattice, centroid of packing shifted to the origin (fig. 2)

Table 2: Results for 2D and 3D constellations ($s = 1$)

Constellation	2D ($e_{ov} = 1.50$)		3D ($e_{ov} = 1.33$)	
	d^2_{min}	Loss [dB]	d^2_{min}	Loss [dB]
1	1.76	3.57	3.10	1.10
2	2.54	1.98	3.20	0.97
3	2.58	1.90	3.45	0.61
4	2.67	1.76	3.56	0.51
5	2.78	1.58	3.63	0.42

C. *Four- and five-dimensional subspaces* ($e_{ov} = 1.25,\ 1.20$)

The densest lattice in 4- and 5-dimensional cases is so-called checkerboard lattice where the coordinates of sphere centers add up to an even integers [14]. However, it is interesting to see that the densest packing is no more optimal from the minimum energy point of view (see (1)) under small oversaturation efficiencies, as was the case with 2D- and 3D-constellations. In fact, in 4D- and 5D-cases the constellation having zero-vector and the rest of signal points of equal energy is better if maximal minimum distance between signal points is a criterion. In 5D-case, even the equal energy constellation is better than the constellation from optimal lattice packing. Another interesting aspect is that according to [8], there is no point to increase subspace dimension to five in the signature per user strategy, since maximal minimum squared distance cannot be greater than $d^2_{\text{min}} = 4$ for $L \geq 4$. However, using collaborative coding in signal design, the minimum distance is improved further when $L \geq 4$. Results for different $(5, 4)$ and $(6, 5)$ constellations are shown in table 3. Equal energy constellations are again taken from [15]. It is seen from the table that the minimum distance is in fact better than with conventional orthogonal signaling (negative loss implies gain). It can be also seen that the effect of balancing is negligible when the number of signal points in constellation is large.

D. *Four- and five-dimensional subspaces having two extra users* ($e_{ov} = 1.50,\ 1.40$)

Results become even more interesting when four- and five-dimensional subspaces are oversaturated by two users ($s = 2$). In case of signature per user approach [8] similar attempt leads to approximately 1 dB higher energy loss than oversaturation of subspaces with only one extra user ($s = 1$). With collaborative coding, the performance can be further enhanced by adding more users to sub-spaces (increasing s), since the energy of one extra user is available to increase minimum distance. Results for $(6,4)$ and $(7,5)$ constellations are illustrated in table 4, where, again, negative loss means gain.

Table 3: Results for 4D and 5D constellations ($s=1$)

Constellation	4D ($e_{ov}=1.25$)		5D ($e_{ov}=1.20$)	
	$d^2_{\min}$	Loss [dB]	$d^2_{\min}$	Loss [dB]
2	4	0	4.36	-0.38
1	4.04	-0.05	4.73	-0.73
4	4.21	-0.22	4.46	-0.48
5	4.22	-0.23	4.47	-0.48
3	4.22	-0.24	4.83	-0.82

Table 4: Results for 4D and 5D constellations ($s=2$)

Constellation	4D ($e_{ov}=1.50$)		5D ($e_{ov}=1.40$)	
	$d^2_{\min}$	Loss [dB]	$d^2_{\min}$	Loss [dB]
1	3.13	1.07	3.92	0.08
3	3.19	0.99	3.97	0.03
2	3.20	0.97	4.15	-0.16
4	3.28	0.86	4.19	-0.20
5	3.28	0.86	4.19	-0.20

V. SUMMARY

Oversaturated S-CDMA system based on group orthogonal signaling and collaborative coding was proposed. Several energy effective signal constellation configurations were designed, providing significantly better energy efficiency (in terms of minimum Euclidean distance) in comparison with signature-per-user oversaturation. The results are summarized in fig. 6, where loss (or gain) of all signal constellations against conventional orthogonal signaling is illustrated versus oversaturation efficiency. Margin between energy efficiency of collaborative coding and signature peruser oversaturation (solid line) is manifested clearly. Collaborative coding allows also flexible trade-offs between oversaturation efficiency, error perfromance and receiver complexity.

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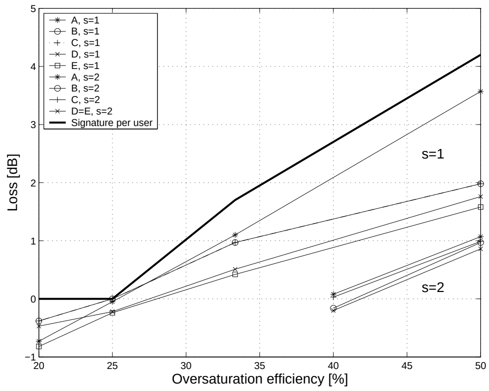

Figure 6: Comparison of results for different signal constellations

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