ELSEVIER

European Journal of Operational Research 85 (1995) 66–81

EUROPEAN
JOURNAL
OF OPERATIONAL
RESEARCH

Theory and Methodology

Financial statement planning in the presence of tax constraints

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Received March 1992; revised June 1993

Abstract

We present a mathematical model of financial statement planning in the case when there is a strong interdependence between taxation and the books. This is the case in Finnish accounting and taxation legislation, making the planning of the financial statements quite complicated for Finnish companies. We present a general model for the planning process and use this model to describe different kinds of situations that companies can find themselves in when developing their financial statements at the end of the year. We also shortly describe how this model is implemented in an expert system for financial statement planning.

Keywords: Financial statement planning; Tax constraints; Optimization; Expert systems; Accounting

1. Introduction

Developing the financial statements in a Finnish company requires a lot of expertise because of the planning dimension in the development process. Possibilities of planning the financial statements are mainly dependent on the way in which the Accounting Act, the Companies Act and a number of tax laws are connected.

On the one hand, the net profit in the financial statements forms the basis for deciding how much dividend can be paid, how much can be given in loan to the shareholders and other partners, and how much can be transferred to the equity capital of the company.

On the other hand, the net profit also forms the basis for calculating the taxes for the accounting period. The tax laws permit a company to use certain income smoothing instruments to reduce its taxes. A number of reserves can be created and charged against taxable income. Depreciation is permitted to be higher in taxation than that based on the estimated useful life of the asset. Such depreciations are, however, accepted as deductions in taxation only if they are also made in the books, to at least the same accumulated amount as in taxation. There is also a long list of items which are treated differently in taxation and in the books.

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SSDI 0377-2217(93)E0229-Q

This means that the financial statement planner is faced with trying to achieve two opposing goals: to show a high net profit and to pay as little tax as possible. Finnish legislation provides considerable scope for planning, and many different issues have to be considered in order to establish an optimal balance between these two goals.

There are also many other complications in the financial statement planning process. The creation of reserves and/or the existence of tax-exempt income can, for example, lead to such a low taxable income that the company will become taxed according to the Tax Board's estimate, i.e., the Tax Board determines the taxable income. This is usually a certain percentage of turnover. Too low a taxable income can also lead to additional taxes if the company pays dividends.

The financial statement process is important for the company, with substantial economic risk involved and therefore management usually takes part in it actively. Choosing between a multitude of possible alternatives, handling the chosen alternative in the best possible way, lack of expertise and cost of expertise are typical problems encountered in financial statement planning. These are all aspects where computer support could increase both effectiveness and efficiency and lead to the development of the expert system Finstex [1], which supports accountants in developing financial statements.

In developing Finstex, it turned out that the core problem in planning financial statements could be described as an optimization model. In analyzing different situations under varying constraints the model also revealed anomalous situations in the financial statement planning that the practitioners were not aware of.

An empirical test [2] conducted with practitioners showed not only that the practitioners have difficulties in knowing the options for using different reserves, and what items are treated differently in the books than in the taxation, but also that they are unaware of the optimization model, or at least they did not use it in the planning process. Instead optimization was sought by trial and error. The experiment showed that practitioners could be quite far off from an optimal solution. Moreover, it showed that the more inexperienced a practitioner is the bigger the difficulties are in formulating the goal, solving the problems in the planning process and achieving acceptable solutions. By using Finstex which contains the mathematical model, another group of practitioners could present considerably better planning results in solving the same task.

Much attention has lately been given to the integration of expert systems and conventional operations research techniques both on the conceptual level (e.g., Turban and Trippi [8]) and on the practical level (e.g., Winkelbauer and Markstrom [9] and Ghiaseddin et al. [5]). Recently also an edited book has been published in this area [3].

Our aim here is to present the mathematical model for financial statement planning that is embedded in Finstex. We give a detailed analysis of different situations that can occur under varying constraints. We do this by considering a collection of problems that the financial statement planner can be faced with. The collection is chosen in order to highlight and explain the effects that can occur and the trade offs that need to be made in planning financial statements.

The rest of the paper is organized as follows: The next section describes the underlying model for financial statement planning. The third section presents and analyses our collection of different financial statement planning situations. The fourth section describes shortly how the model is implemented in Finstex. Section 5 summarizes the paper.

2. A model for financial statement planning

The mathematical model is based on results gained from the knowledge acquisition process used in constructing Finstex.

The purpose of financial statement planning can be stated as follows [7]:
- to fulfil the requirements of the financiers on return of equity capital with as small tax expenses as possible;
- to forecast tax consequences of different income smoothing actions;
- to try to create room in the income statement for future income smoothing actions.

We consider here only the optimization problems associated with financial statement planning and describe how the solutions to them have been implemented in the expert system. The other central question, what income smoothing actions to take and in which order they should be taken is not considered here. A detailed account of this issue is given in [1].

The starting point for financial statement planning is the *result from the trial balance* (RFT). This result can be decreased or increased through *income smoothing* (IS). After income smoothing we get the *result before taxes* (RBT). This result may be the same as the *taxable income* (TI), but very often it is not, because certain revenues are tax-exempt and certain costs are non-deductible expenses in the taxation. Moreover, income which is not revenue in the books may be added and certain expenses not included in the books may be deducted.

The *taxes* (T) for the period are calculated based on the taxable income. The taxable income consists in reality of two parts, i.e., the *municipal taxable income* (TI_m) and the *state taxable income* (TI_s). These are not always the same because some of the *correction items* accepted for *state taxation* (C_s) are not accepted in the *municipal taxation* (C_m) and vice versa. The municipal taxable income must be at least a *sufficient amount* (SA), otherwise a higher *estimated level* (EL) will be used instead when calculating the municipal taxes. If dividend is paid, the *comparable taxes* (T^c) must exceed the *minimum taxes* due to dividend payment (T_d), otherwise the company has to pay *fulfilment taxes* (T_f).

The accounting *profit* for the period (P) is calculated by deducting the taxes from the result before taxes. A negative *free equity capital* (FEC) should be avoided.

The basic quantities that provide input to the planning process are listed in Table 1, and the defined quantities in Table 2, for ease of reference.

2.1. Definitions

The net profit (P) is given by

$$P = \mathrm{RFT} - \mathrm{IS} - T. \tag{1}$$

Equation (1) can be written as

$$P = \mathrm{RBT} - T \tag{2}$$

Table 1
Given quantities

IS_{min}	= minimum income smoothing amount
IS_{max}	= maximum income smoothing amount
RFT	= result from the trial balance
t_m	= municipal tax rate
t_s	= state tax rate
C_m	= correction items in the municipal taxation
C_s	= correction items in the state taxation
SA	= sufficient amount
EL	= estimated level
T_x	= excess taxes from previous periods
D	= dividend
FEC_p	= free equity capital from previous period

Table 2
Defined quantities

P	= the accounting profit (1)
RBT	= result before taxes (3)
T	= taxes (4)
TI_m	= municipal taxable income (9)
TI_s	= state taxable income (10)
T^c	= comparable taxes (12)
T_d	= minimum taxes due to dividend payment (13)
T_f	= fulfilment taxes (14)
FEC	= free equity capital (15)

where

$$\mathrm{RBT} = \mathrm{RFT} - \mathrm{IS}. \tag{3}$$

The amount of taxes T is determined by

$$T = T_m + T_s \tag{4}$$

where

$$T_m = \begin{cases} t_m * \mathrm{TI}_m & \text{if } \mathrm{TI}_m \geq 0, \\ 0, & \text{otherwise}, \end{cases} \tag{5}$$

and

$$T_s = \begin{cases} t_s * \mathrm{TI}_s & \text{if } \mathrm{TI}_s \geq 0, \\ 0, & \text{otherwise}. \end{cases} \tag{6}$$

The tax rates are non-negative and less than 0.5 each, so we have

$$0 \leq t_m < 0.5, \tag{7}$$

$$0 \leq t_s < 0.5. \tag{8}$$

TI_m is defined as

$$\mathrm{TI}_m = \begin{cases} \mathrm{RBT} - C_m & \text{if } \mathrm{RBT} - C_m \geq \mathrm{SA}, \\ \mathrm{EL} & \text{if } \mathrm{RBT} - C_m < \mathrm{SA}, \end{cases} \tag{9}$$

and TI_s is defined as

$$\mathrm{TI}_s = \mathrm{RBT} - C_s. \tag{10}$$

Comparable taxes are determined as

$$T^c = T_m^c + T_s \tag{11}$$

where

$$T_m^c = \begin{cases} 0.17 * (\mathrm{RBT} - C_m) & \text{if } \mathrm{RBT} - C_m \geq 0, \\ 0, & \text{otherwise}. \end{cases} \tag{12}$$

The minimum taxes due to dividend payment are

$$T_d = \begin{cases} \frac{2}{3} * D - T_x & \text{if } \frac{2}{3} * D - T_x \geq 0, \\ 0, & \text{otherwise}. \end{cases} \tag{13}$$

Fulfilment taxes are determined as

$$T_f = T_d - T^c. \tag{14}$$

The free equity capital is determined as

$$\mathrm{FEC} = \mathrm{FEC}_p + P - D. \tag{15}$$

2.2. Constraints

The margin for income smoothing can vary between a minimum amount $\mathrm{IS}_{\min}$ and a maximum amount $\mathrm{IS}_{\max}$:

$$\mathrm{IS}_{\min} \le \mathrm{IS} \le \mathrm{IS}_{\max}. \tag{16}$$

This can also be described as a restriction on RBT:

$$\mathrm{RBT}_{\min} \le \mathrm{RBT} \le \mathrm{RBT}_{\max} \tag{17}$$

where

$$\mathrm{RBT}_{\min} = \mathrm{RFT} - \mathrm{IS}_{\max} \quad \text{and} \quad \mathrm{RBT}_{\max} = \mathrm{RFT} - \mathrm{IS}_{\min}.$$

Taxation according to the Tax Board's estimate is avoided if

$$\mathrm{RBT} - C_m \ge \mathrm{SA}. \tag{18}$$

If dividend (D) is paid, then free equity capital must be non-negative:

$$\mathrm{FEC} \ge 0. \tag{19}$$

FEC should in normal circumstances be non-negative also when dividend is not paid.

Fulfilment taxes are avoided if

$$T^c \ge T_d. \tag{20}$$

2.3. Goals of planning

The general goal is either to minimize taxes T or to minimize $|G - P|$, the distance of the net profit from certain goal result G which has been set by the planner.

3. Analyzing the goals

Below we consider a collection of real-world problems under various constraints. The presentation starts with less complicated situations and ends with the most difficult ones.

3.1. Problem 1

We want to minimize taxes for a company that is not subject to taxation according to the Tax Board's estimate. This is the case for all companies that have been less than five years in business. No dividend will be paid so we do not have to consider the restriction on free equity capital and the possibility of fulfilment taxes. Thus, the problem is the following:

$$\begin{array}{ll} \text{Minimize} & T = T_m + T_s \\ \text{when} & \mathrm{RBT}_{\min} \le \mathrm{RBT} \le \mathrm{RBT}_{\max}. \end{array}$$

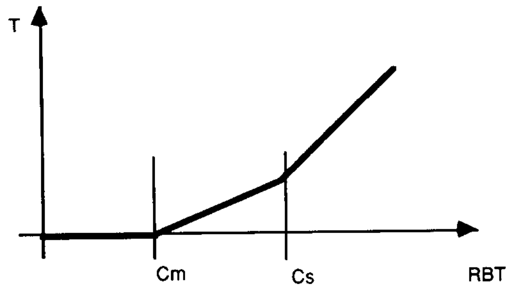

Fig. 1. Taxes as a function of RBT.

We have in this case that

$$T_m = \begin{cases} t_m * (\mathrm{RBT} - C_m) & \text{if RBT} \geq C_m, \\ 0, & \text{otherwise.} \end{cases}$$

Similarly,

$$T_s = \begin{cases} t_s * (\mathrm{RBT} - C_s) & \text{if RBT} \geq C_s, \\ 0, & \text{otherwise.} \end{cases}$$

Let us assume that $C_m \leq C_s$. Then we have that T is defined piecewise:

$$T = \begin{cases} (t_m + t_s) * \mathrm{RBT} - t_m * C_m - t_s * C_s & \text{if RBT} \geq C_s, \\ t_m * \mathrm{RBT} - t_m * C_m & \text{if } C_m \leq \mathrm{RBT} < C_s, \\ 0 & \text{if RBT} < C_m. \end{cases}$$

Taxes T as a function of RBT is shown in Fig. 1.

Consider now the constraint on RBT. We have two possible cases for choosing an optimum value of RBT to minimize the taxes T.

(a) If $\mathrm{RBT}_{min} < \mathrm{C}_m$, then any RBT such that $\mathrm{RBT}_{min} \leq \mathrm{RBT} < \mathrm{C}_m$ gives an optimum solution. In this case we choose the largest optimum, $\mathrm{RBT}^{\circ} = \mathrm{C}_m$.

(b) Otherwise $\mathrm{RBT}_{min} \geq C_m$. In this case we obviously have that optimum is $\mathrm{RBT}^{\circ} = \mathrm{RBT}_{min}$.

Example 1. Let us assume the following initial data:
The result from the trial balance: RFT = 15000.
The amount by which RFT can be decreased: $\mathrm{IS}_{max} = 15000$.
The amount by which RFT can be increased: $\mathrm{IS}_{min} = -5000$.
Correction items accepted for municipal taxation: $C_m = 500$.
Correction items accepted for state taxation: $C_s = 700$.
Municipal tax rate: $t_m = 0.17$.
State tax rate: $t_s = 0.25$.

Then $\mathrm{RBT}_{max} = 20000$ and $\mathrm{RBT}_{min} = 0$. The optimum solution RBT° is given by case (a) above, $\mathrm{RBT}^{\circ} = 500$ and $T^{\circ} = 0$. Example 1 reveals that using all possible income smoothing (15000) will not decrease our taxes for the period. They only lead to unnecessary use of income smoothing by an amount of 500, which might be better needed next year.

Example 2. Let the conditions be as in Example 1, except that $IS_{max} = 10000$. Then $RBT_{max} = 20000$, $RBT_{min} = 5000$ and the optimum solution $RBT^o = 5000$ is given by case (b). The company has not enough income smoothing available and therefore pays taxes amounting to 1840 for the period ($T_m + T_s = 0.17 * 4500 + 0.25 * 4300 = 1840$).

3.2. Problem 2

We change the previous problem so that the company may be subject to taxation according to the Tax Board's estimate. The company is still not paying dividends which means that the restriction on free equity capital and the possibility of fulfilment taxes need not be considered. We assume for simplicity that $C_s \leq RBT_{min}$ so that state taxes may not become zero. Thus, the problem is to

$$\begin{array}{ll} \text{Minimize} & T = T_m + T_s \\ \text{when} & RBT_{min} \leq RBT \leq RBT_{max}. \end{array}$$

Let us first look at T_m. Using the definition of TI_m, we have that

$$TI_m = \begin{cases} RBT - C_m & \text{if } RBT \geq SA + C_m, \\ EL & \text{if } RBT < SA + C_m. \end{cases}$$

Because both $SA > 0$ and $EL > 0$, we have that $T_m = t_m * TI_m$. Because of the assumption that state taxes may not become zero, we also have that $T_s = t_s * (RBT - C_s)$. This gives us two cases in the definition of T:

$$T = \begin{cases} t_m * (RBT - C_m) + t_s * (RBT - C_s) & \text{if } RBT \geq SA + C_m, \\ t_m * EL + t_s * (RBT - C_s) & \text{if } RBT < SA + C_m. \end{cases}$$

Rearranging the terms gives us then

$$T = \begin{cases} (t_m + t_s) * RBT - t_m * C_m - t_s * C_s & \text{if } RBT \geq SA + C_m, \\ t_s * RBT + t_m * EL - t_s * C_s & \text{if } RBT < SA + C_m. \end{cases}$$

Taxes as a function of RBT are shown in Fig. 2.

We can analyze this figure as follows. Let $RBT = H$ be the solution to the equation

$$t_s * RBT + t_m * EL - t_s * C_s = (t_m + t_s) * (SA + C_m) - t_m * C_m - t_s * C_s.$$

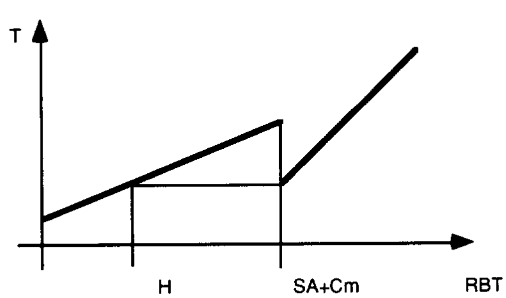

Fig. 2. Tax T as a function of RBT.

The point H is the place where it is equally favorable to be taxed according to the Tax Board's estimate as it is immediately before the company becomes taxed according to this (see Fig. 2).

We now have the following two cases, when choosing the optimum value for RBT:

(a) If $H < \text{RBT}_{\text{min}} < \text{SA} + C_{\text{m}}$ and $\text{RBT}_{\text{max}} \geq \text{SA} + C_{\text{m}}$, then it is more favorable to avoid taxation according to the Tax Board's estimate, so we choose $\text{RBT}^{\circ} = \text{SA} + C_{\text{m}}$.

(b) Otherwise we choose $\text{RBT}^{\circ} = \text{RBT}_{\text{min}}$.

Example 3. Let us again assume the conditions of Example 1, except that this time $\text{IS}_{\text{max}} = 9000$. Additionally we have that $\text{EL} = 7000$ and $\text{SA} = 5600$.

Then $\text{RBT}_{\text{max}} = 20000$, $\text{RBT}_{\text{min}} = 6000$ and $H = 5148$. In choosing the optimal RBT° we use case (a) and choose $\text{RBT}^{\circ} = \text{SA} + C_{\text{m}} = 6100$, resulting in taxes T° for the period amounting to 2302 ($T_{\text{m}} = 952$ and $T_{\text{s}} = 1350$). If the company would choose to use all income smoothing, it would result in taxation according to the Tax Board's estimate and higher taxes for the period ($T = 2515$, where $T_{\text{m}} = 1190$ and $T_{\text{s}} = 1325$), i.e., the lower state taxes cannot compensate for the higher municipal taxes. Moreover, the company would use unnecessary reserves that could be used instead in the next period to decrease the taxable income.

Example 4. Let us assume that we again have the conditions of Example 3, except that this time $\text{IS}_{\text{max}} = 10000$.

Then $\text{RBT}_{\text{max}} = 20000$, $\text{RBT}_{\text{min}} = 5000$ and $H = 5148$. In choosing the optimal RBT° we use case (b) and choose $\text{RBT}^{\circ} = \text{RBT}_{\text{min}} = 5000$, resulting in taxes for the period amounting to 2265 ($T_{\text{m}} = 1190$ and $T_{\text{s}} = 1075$), i.e., in this case we have moved away from the unfavorable area in Fig. 2. If a company would instead choose to avoid taxation according to the Tax Board's estimate, that would result in higher taxes for the period, amounting to 2302 ($T_{\text{m}} = 952$ and $T_{\text{s}} = 1350$).

3.3. Problem 3

Consider next the previous problem, when the company may be subject to taxation according to the Tax Board's estimate, but now permitting state taxes to become zero. The restriction on free equity capital and the possibility of fulfilment taxes are still not considered. Thus, the problem is to

$$\begin{array}{ll} \text{Minimize} & T = T_{\text{m}} + T_{\text{s}} \\ \text{when} & \text{RBT}_{\text{min}} \leq \text{RBT} \leq \text{RBT}_{\text{max}}. \end{array}$$

Let us first look at T_{m}. Using the definition of TI_{m}, we have that

$$\text{TI}_{\text{m}} = \begin{cases} \text{RBT} - C_{\text{m}} & \text{if } \text{RBT} \geq \text{SA} + C_{\text{m}}, \\ \text{EL} & \text{if } \text{RBT} < \text{SA} + C_{\text{m}}. \end{cases}$$

Because both $\text{SA} > 0$ and $\text{EL} > 0$, we have that $T_{\text{m}} = t_{\text{m}} * \text{TI}_{\text{m}}$. As T_{s} is defined as before, this gives us four cases in the definition of T:

$$T = \begin{cases} t_{\text{m}} * (\text{RBT} - C_{\text{m}}) + t_{\text{s}} * (\text{RBT} - C_{\text{s}}) & \text{in case A}, \\ t_{\text{m}} * (\text{RBT} - C_{\text{m}}) & \text{in case B}, \\ t_{\text{m}} * \text{EL} + t_{\text{s}} * (\text{RBT} - C_{\text{s}}) & \text{in case C}, \\ t_{\text{m}} * \text{EL} & \text{in case D}. \end{cases}$$

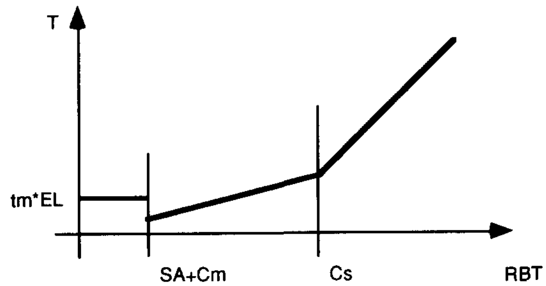

Fig. 3. Tax T as a function of RBT, when $\mathrm{SA} + C_m \le C_s$.

Here the cases are as follows:

Case A: $\mathrm{RBT} \ge \mathrm{SA} + C_m$ and $\mathrm{RBT} \ge C_s$,
Case B: $\mathrm{RBT} \ge \mathrm{SA} + C_m$ and $\mathrm{RBT} < C_s$,
Case C: $\mathrm{RBT} < \mathrm{SA} + C_m$ and $\mathrm{RBT} \ge C_s$,
Case D: $\mathrm{RBT} < \mathrm{SA} + C_m$ and $\mathrm{RBT} < C_s$.

Rearranging the terms gives us then

$$T = \begin{cases} (t_m + t_s) * \mathrm{RBT} - t_m * C_m - t_s * C_s & \text{in case A,} \\ t_m * \mathrm{RBT} - t_m * C_m & \text{in case B,} \\ t_s * \mathrm{RBT} + t_m * \mathrm{EL} - t_s * C_s & \text{in case C,} \\ t_m * \mathrm{EL} & \text{in case D.} \end{cases}$$

We have two possibilities: Either $\mathrm{SA} + C_m < C_s$ or $C_s \le \mathrm{SA} + C_m$. The situation when $\mathrm{SA} + C_m < C_s$ holds is shown in Fig. 3. The optimum is determined as follows:
(a) If $\mathrm{RBT}_{max} < \mathrm{SA} + C_m$, then any RBT in the permitted interval is optimum. We choose $\mathrm{RBT}^\circ = \mathrm{RBT}_{max}$. Then $T^\circ = t_m * \mathrm{EL}$.
(b) If $\mathrm{RBT}_{min} > \mathrm{SA} + C_m$, then we choose $\mathrm{RBT}^\circ = \mathrm{RBT}_{min}$.
(c) If neither of the above holds, then $\mathrm{RBT}_{min} \le \mathrm{SA} + C_m \le \mathrm{RBT}_{max}$. In that case, we should avoid taxation according to the Tax Board's estimate, so we choose $\mathrm{RBT}^\circ = \mathrm{SA} + C_m$.

The situation when $C_s < \mathrm{SA} + C_m$ holds is shown in Fig. 4. The optimum is determined as follows:
(a) If $\mathrm{RBT}_{max} < C_s$, then any RBT in the permitted interval is optimum. We choose $\mathrm{RBT}^\circ = \mathrm{RBT}_{max}$.

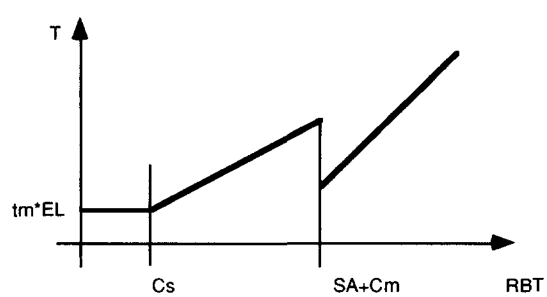

Fig. 4. Tax T as a function of RBT, when $C_s < \mathrm{SA} + C_m$.

Then $T^\circ = t_m * \text{EL}$.
(b) If $\text{RBT}_{min} > \text{SA} + C_m$, then we choose $\text{RBT}^\circ = \text{RBT}_{min}$.
(c) If $C_s \le \text{RBT}_{max} < \text{SA} + C_m$, then we choose $\text{RBT}^\circ = \max\{\text{RBT}_{min}, C_s\}$.
(d) In the last case we have that $\text{SA} + C_m \le \text{RBT}_{max}$ and $\text{RBT}_{min} \le \text{SA} + C_m$. In this case, the unfavorable part of the curve is within this interval, and we have to avoid this, if possible. Then we choose for RBT° either $\max\{\text{RBT}_{min}, C_s\}$ or $\text{SA} + C_m$, whichever gives the smaller tax value.

Example 5. Let us assume that we again have the conditions of Example 1, except that this time $\text{IS}_{max} = 10000$, $C_x = 6500$ and additionally that $\text{EL} = 7000$ and $\text{SA} = 5600$.

Then $H = 5148$, $\text{RBT}_{max} = 20000$, $\text{RBT}_{min} = 5000$ and $\text{SA} + C_m = 5600 + 500 < C_s = 6500$. Because $\text{RBT}_{min} = 5000 < \text{SA} + C_m = 6100 < \text{RBT}_{max} = 20000$, the optimal solution $\text{RBT}^\circ = \text{SA} + C_m = 6100$ is given by (c), which gives $T^\circ = 952$. If an accountant in this case uses maximum income smoothing he will again pay more taxes (1190) than necessary and again use unnecessary income smoothing that could have been saved for the next year.

Example 6. Assumptions are the same as in Example 5 but we change C_s to 6000, i.e., we now have the situation that $C_s = 6000 < \text{SA} + C_m = 6100$. The optimal value is given by (d): $\text{RBT}^\circ = \text{SA} + C_m = 6100$, because this value gives the smaller tax value $T^\circ = 977$.

3.4. Problem 4

Consider now the problem of reaching a certain predefined goal result G. The restrictions on free equity capital and the possibility of fulfilment taxes are not considered. Also, for simplicity, we assume that neither state nor municipal taxes can become zero. Thus, the problem is the following:

$$\begin{array}{ll} \text{Minimize} & |G - P| \\ \text{when} & \text{RBT}_{min} \le \text{RBT} \le \text{RBT}_{max}, \\ & C_m, C_s \le \text{RBT}_{min}, \end{array}$$

We then have that

$$T = t_m * (\text{RBT} - C_m) + t_s * (\text{RBT} - C_s).$$

Hence, by the definition of P, the task is to minimize

$$|G - \text{RBT} + t_m * (\text{RBT} - C_m) + t_s * (\text{RBT} - C_s)|.$$

Rearranging this gives us the cost function

$$|G - t_m * C_m - t_s * C_s - (1 - t_m - t_s) * \text{RBT}|.$$

We can write this as a piecewise linear function:

$$F = \begin{cases} K - h * \text{RBT} & \text{if RBT} \le K/h, \\ -K + h * \text{RBT} & \text{if RBT} > K/h, \end{cases}$$

where $K = G - t_m * C_m - t_s * C_s$ and $h = 1 - t_m - t_s$. The function F as a function of RBT is shown in Fig. 5.

Let $\text{RBT} = R$ be the solution to the equation

$$K - h * \text{RBT} = 0,$$

i.e., $R = K/h$.

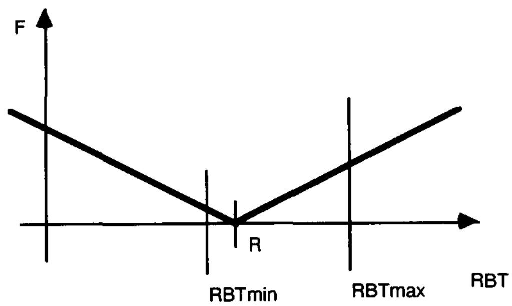

Fig. 5. Trying to reach a certain goal.

Consider now the constraint on RBT. We have three possible cases for the optimal solution:
(a) If $\text{RBT}_{\max} < R$, then $\text{RBT}^{\circ} = \text{RBT}_{\max}$.
(b) If $\text{RBT}_{\min} > R$, then $\text{RBT}^{\circ} = \text{RBT}_{\min}$.
(c) Otherwise $\text{RBT}_{\min} \le R \le \text{RBT}_{\max}$, and the optimal solution is $\text{RBT}^{\circ} = R$, for which $G = P$ holds.

3.5. Problem 5

We again want to minimize taxes for a company that is not subject to taxation according to the Tax Board's estimate. We assume that neither state nor municipal taxes will become zero. We now extend the previous treatment to also consider the restrictions on free equity capital and the need to avoid fulfilment taxes. Thus, the problem is to

$$
\begin{array}{ll}
\text{Minimize} & T = T_{\mathrm{m}} + T_{\mathrm{s}} \\
\text{when} & \text{RBT}_{\min} \le \text{RBT} \le \text{RBT}_{\min}, \\
& C_{\mathrm{m}}, C_{\mathrm{s}} \le \text{RBT}_{\max}, \\
& \text{FEC}_{\mathrm{p}} + P - D \ge 0, \\
& T^{\mathrm{c}} \ge T_{\mathrm{d}}.
\end{array}
$$

We have in this case that

$$T = t_{\mathrm{m}} * (\text{RBT} - C_{\mathrm{m}}) + t_{\mathrm{s}} * (\text{RBT} - C_{\mathrm{s}}).$$

The restriction on free equity capital is equivalent to

$$\text{FEC}_{\mathrm{p}} + \text{RBT} - T - D \le 0.$$

Solving this with the above definition of T gives the following restriction:

$$\text{FEC}_{\mathrm{p}} + \text{RBT} - (t_{\mathrm{m}} + t_{\mathrm{s}}) * \text{RBT} + t_{\mathrm{m}} * C_{\mathrm{m}} + t_{\mathrm{s}} * C_{\mathrm{s}} - D \ge 0.$$

Rearrangement gives the restriction

$$\text{RBT} \ge R,$$

where $R = (D - \text{FEC}_{\mathrm{p}} - t_{\mathrm{m}} * C_{\mathrm{m}} - t_{\mathrm{s}} * C_{\mathrm{s}})/(1 - t_{\mathrm{m}} - t_{\mathrm{s}})$.
Fulfillment taxes are avoided if

$$T^{\mathrm{c}} \ge T_{\mathrm{d}}.$$

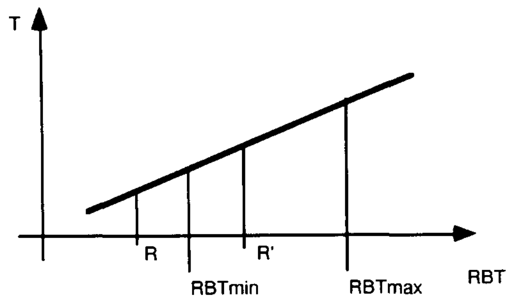

Fig. 6. Constraints on equity capital and comparable taxes.

The value of T_d is fixed, but T^c depends on RBT. By definition (11), we then get the following restriction:

$$(0.17 + t_s) * \mathrm{RBT} - 0.17 * C_m - t_s * C_s \geq T_d.$$

Rearrangement gives us that

$$\mathrm{RBT} \geq R'$$

where $R' = (T_d + 0.17 * C_m + t_s * C_s)/(0.17 + t_s)$. Thus, the additional constraint can be added as constraints on the range of permitted RBT. We get the restriction that

$$R'' \leq \mathrm{RBT} \leq \mathrm{RBT}_{max},$$

where $R'' = \max\{\mathrm{RBT}_{min}, R, R'\}$

The optimum is easily seen from Fig. 6. We have that $\mathrm{RBT}^o = R''$. This result requires that $\mathrm{RBT}_{max} \geq R''$. Otherwise there is no feasible solution to the problem. This means that either fulfilment taxes cannot be avoided, or the company cannot satisfy the restriction on free equity capital being non-negative.

Example 7. Let us assume that we again have the conditions of Example 1, except that this time the company pays a dividend D amounting to 3000. Moreover, $\mathrm{FEC}_p = 2000$ and $T_x = 0$.

Then $\mathrm{RBT}_{min} = 0$, $\mathrm{RBT}_{max} = 20000$, $R = 1276$, $R' = 5381$, and $R'' = 5381$, i.e., $\mathrm{RBT}^o = 5381$ and $T^o = 2000$. By using all income smoothing the company would instead this time have to pay fulfilment taxes and would again have used unnecessary income smoothing.

3.6. Problem 6

We want again to minimize taxes for a company that is not subject to taxation according to the Tax Board's estimate, but this time we permit both state and municipal taxes to become zero, and we also consider the restrictions on free equity capital and the need to avoid fulfilment taxes. Thus, the problem is the following:

$$\begin{aligned} \text{Minimize} \quad & T = T_m + T_s \\ \text{when} \quad & \mathrm{RBT}_{min} \leq \mathrm{RBT} \leq \mathrm{RBT}_{max}, \\ & \mathrm{FEC}_p + P - D \geq 0, \\ & T^c \geq T_d. \end{aligned}$$

We have in this case that

$$T_m = \begin{cases} t_m * (\mathrm{RBT} - C_m) & \text{if } \mathrm{RBT} \geq C_m, \\ 0, & \text{otherwise.} \end{cases}$$

Similarly,

$$T_s = \begin{cases} t_s * (\mathrm{RBT} - C_s) & \text{if } \mathrm{RBT} \geq C_s, \\ 0, & \text{otherwise.} \end{cases}$$

Let us assume that $C_m \leq C_s$. Then we have that T is defined piecewise;

$$T = \begin{cases} (t_m + t_s) * \mathrm{RBT} - t_m * C_m - t_s * C_s & \text{if } \mathrm{RBT} \geq C_s, \\ t_m * \mathrm{RBT} - t_m * C_m & \text{if } C_m \leq \mathrm{RBT} < C_s, \\ 0 & \text{if } \mathrm{RBT} < C_m. \end{cases}$$

Taxes T as a function of RBT is shown in Fig. 1.

The restriction on free equity capital is

$$\mathrm{FEC} = \mathrm{FEC}_p + P - D \geq 0.$$

By (2), we have that this is equivalent to

$$\mathrm{FEC}_p + \mathrm{RBT} - T - D \geq 0.$$

Solving this using the above definition of T gives the following restriction:

$$\begin{array}{ll} \mathrm{FEC}_p + \mathrm{RBT} - (t_m + t_s) * \mathrm{RBT} + t_m * C_m + t_s * C_s - D \geq 0 & \text{if } \mathrm{RBT} \geq C_s, \\ \mathrm{FEC}_p + \mathrm{RBT} - t_m * \mathrm{RBT} + t_m * C_m - D \geq 0 & \text{if } C_m \leq \mathrm{RBT} < C_s, \\ \mathrm{FEC}_p + \mathrm{RBT} - 0 - D \geq 0 & \text{if } \mathrm{RBT} < C_m. \end{array}$$

Simplifying these constraints gives the restrictions

$$\begin{array}{ll} \mathrm{RBT} \geq R_1 & \text{if } \mathrm{RBT} \geq C_s, \\ \mathrm{RBT} \geq R_2 & \text{if } C_m \leq \mathrm{RBT} < C_s, \\ \mathrm{RBT} \geq R_3 & \text{if } \mathrm{RBT} < C_m, \end{array}$$

where

$$R_1 = (D - \mathrm{FEC}_p - t_m * C_m - t_s * C_s)/(1 - t_m - t_s),$$

$$R_2 = (D - \mathrm{FEC}_p - t_m * C_m)/(1 - t_m),$$

$$R_3 = D - \mathrm{FEC}_p.$$

Fulfillment taxes are avoided if the following condition holds:

$$T^c \geq T_d.$$

The value of T_d is fixed, but T^c depends on RBT. By definition (11), we again have the following restriction:

$$\begin{array}{ll} (0.17 + t_s) * \mathrm{RBT} - 0.17 * C_m - t_s * C_s \geq T_d & \text{if } \mathrm{RBT} \geq C_s, \\ 0.17 * \mathrm{RBT} - 0.17 * C_m \geq T_d & \text{if } C_m \leq \mathrm{RBT} < C_s, \\ 0 \geq T_d & \text{if } \mathrm{RBT} < C_m. \end{array}$$

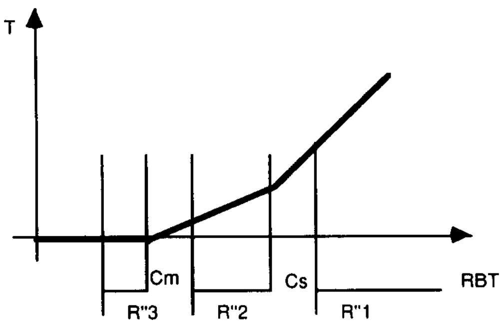

Fig. 7. Constraints on equity capital and comparable taxes, zero taxes permitted.

Simplifying this restriction gives us that

$$\begin{array}{ll} \mathrm{RBT} \geq R'_1 & \text{if } \mathrm{RBT} \geq C_s, \\ \mathrm{RBT} \geq R'_2 & \text{if } C_m \leq \mathrm{RBT} < C_s, \\ T_d = 0 & \text{if } \mathrm{RBT} < C_m, \end{array}$$

where

$$R'_1 = (T_d + 0.17 * C_m + t_s * C_s)/(0.17 + t_s),$$

$$R'_2 = (t_d + 0.17 * C_m)/0.17.$$

Thus, in each of the three intervals used in defining T, we have additional restrictions that must be met. Combining these restrictions gives us the following general restriction:

$$\begin{array}{ll} R''_1 \leq \mathrm{RBT} \leq \mathrm{RBT}_{max} & \text{if } \mathrm{RBT} \geq C_s \\ R''_2 \leq \mathrm{RBT} \leq \mathrm{RBT}_{\max} & \text{if } C_m \leq \mathrm{RBT} < C_s \\ R''_3 \leq \mathrm{RBT} \leq \mathrm{RBT}_{\max} & \text{and} \quad T_d = 0 \quad \text{if } \mathrm{RBT} < C_m, \end{array}$$

where

$$R''_1 = \max\{\mathrm{RBT}_{\min}, R_1, R'_1\},$$

$$R''_2 = \max\{\mathrm{RBT}_{\min}, R_2, R'_2\},$$

$$R''_3 = \max\{\mathrm{RBT}_{\min}, R_3\}.$$

The optimum in this case is again easily seen from Fig. 7:

(a) If the restriction $R''_3 \leq \mathrm{RBT} \leq \mathrm{RBT}_{\max}$ and $T_d = 0$ holds for some RBT such that RBT < C_m, then any such RBT is an optimal solution (and $T^\circ = 0$).
(b) Otherwise, if the restriction $R''_2 \leq \mathrm{RBT} \leq \mathrm{RBT}_{\max}$ holds for some RBT such that $C_m \leq \mathrm{RBT} < C_s$, then the optimum is $\mathrm{RBT}^\circ = R''_2$.
(c) Otherwise, if the restriction $R''_1 \leq \mathrm{RBT} \leq \mathrm{RBT}_{\max}$ holds for some RBT such that RBT ≥ C_s, then the optimum is $\mathrm{RBT}^\circ = R''_1$.
(d) If none of the previous cases hold, then there is no feasible solution to the problem. This means like in Problem 5 that either fulfilment taxes cannot be avoided, or the company cannot satisfy the restriction on free equity capital being non-negative.

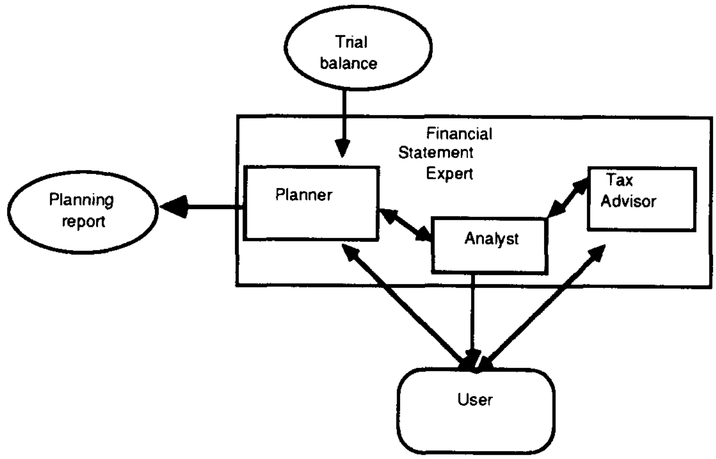

Fig. 8. Finstex.

4. Implementation of the model in Finstex

Finstex consists of three main parts, Planner, Analyst and Tax Advisor (see Fig. 8). The system is written in Prolog and consists of about 800 rules.

Planner is the central part of the system. It contains a menu from which the user can choose a goal for the planning. Planner has explicit knowledge about the relevant tax laws. It knows about the available income smoothing instruments and who is entitled to them, i.e., it helps the user to determine IS_{max} and IS_{min}. If the company cannot use all income smoothing opportunities Planner helps the user in determining in what order these should be used to minimize the taxes within a planning range of two to three years, (e.g., should the company use the inventory reserve before maximum tax depreciation or vice versa). Planner can also perform 'what if' analyses if the user so wishes.

Analyst has knowledge about how to analyze the financial statements situation for the company. It analyses and reports whether the planning goal can be achieved or not. Analyst will also report when the goal has been set so low that the company will be taxed according to the Tax Board's estimate ($RBT - C_m < SA$, constraint (18)), or when the company will be forced to pay fulfilment taxes ($T^c < T_d$, constraint (20)). Analyst contains the optimization algorithm, which is implemented in Prolog.

Tax Advisor knows about the items treated differently in taxation and in the books. It effectively assists the user in determining the correction items, C_s and C_m, by carrying out a dialog with the user, asking questions about the specific situation that the company is in, and answering questions about the correction items and what statutes are applicable in each situation. The specific correction items that a company is entitled to is very much dependent on the specifics of the company, and requires an elaborate analysis of what different tax laws may be applied in the situation at hand, and what options are available to the planner. Tax Advisor has a large rule base, containing those parts of statutes and laws that are relevant to financial statement planning.

5. Summary

The financial statement process in Finnish companies is a central activity with many people involved in the planning phase. It is an important task, where mistakes can be very costly for the company. A practitioner developing the financial statements for a company would not typically formulate a mathematical model when developing the financial statements for the accounting period.

In this paper we have formulated the financial statement planning problem as an optimization problem, and have analysed a number of different situations that can arise in practice. This analysis shows that the optimum can be rather complicated to find, and explains why it can be difficult for practitioners to come up with an optimal solution for the period by trial and error.

The model for the financial statement analysis has been embedded in an expert system that aids the user in identifying and determining the parameters needed by the model. The general problem of finding an optimum in the financial statement planning depends as much on a proper determination of the parameters involved as on the correct solution of the mathematical model presented here. Both of these tasks are quite difficult for practitioners, who make a lot of errors in the planning process, and seldom actually find a fully optimal solution. In practical experiments [2], the expert system support proved to increase the quality of the planning process substantially, and also eliminated most of the errors in the planning process.

Acknowledgments

The authors gratefully acknowledge P. Rubin and the anonymous referees for their comments on the paper. Part of the work for this paper was carried out during a sabbatical leave of the first author at University of Southern California and of the second author at the California Institute of Technology.

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