Soft computing and the bullwhip effect*

Christer Carlsson and Robert Fullér[†]

IAMSR, Åbo Akademi University,
Lemminkainengatan 14B, FIN-20520 Åbo, Finland
e-mail: christer.carlsson, robert.fuller@ra.abo.fi

Abstract: We consider a series of companies in a supply chain, each of which orders from its immediate upstream members. Usually, the retailer's order do not coincide with the actual retail sales. The bullwhip effect refers to the phenomenon where orders to the supplier tend to have larger variance than sales to the buyer (i.e. demand distortion), and the distortion propagates upstream in an amplified form (i.e. variance amplification). We show that if the members of the supply chain share information, and agree on better and better fuzzy estimates (as time advances) on future sales for the upcoming period, then the bullwhip effect can be essentially reduced.

Keywords: supply chain, bullwhip effect, fuzzy number, variance of fuzzy numbers

1. Introduction

The *Bullwhip Effect* has been the focus of systematic theoretical work only in recent years. The first articles to report on research results in a more systematic fashion [5] have been published only recently. The effect is often identified with the simulation experiment, The Beer Game, which is used to demonstrate the effects of distorted in- formation in the supply chain (which is the cause of the bullwhip effect).

There are some examples published which demonstrate the bullwhip effect: (i) P & G has over the years been successful producers and sellers of Pampers, and they have seen that babies are reliable and steady consumers; (ii) the retailers, however, show fluctuating sales, although the demand should be easy to estimate as soon as the number of babies is known; (iii) P & G found out that the orders they received from distributors showed a strong variability, in fact much stronger than could be

Economics & Complexity

Vol. 2 N. 3

Winter 1999-00 - p. 1-26

explained by the fluctuating sales of the retailers; finally, (iv) when P &G studied their own orders to 3M for raw material they found these to be wildly fluctuating, actually much more than could be explained by the orders from the distributors.

The context we have chosen for this study is the forest products industry and the markets for fine paper products. The supply chain is thus a business-to-business chain, and we will show that the bullwhip effect is as dominant as in the business-to-consumer supply chain.

The key driver of the bullwhip effect appears to be that the variability of the estimates or the forecasts of customer demand seems to amplify as the orders move up the supply chain from the customer, through retailers and wholesalers to the producer of the product or service. This is called the *bullwhip*, the whiplash or the whipsaw effect.

In a number of studies, it appears that the bullwhip effect will have a number of negative effects and that it will cause significant inefficiencies:

1. Excessive inventory investments throughout the supply chain as retailers, distributors, logistics operators and producers need to safeguard themselves against the variations.
2. Poor customer service as some part of the supply chain runs out of products due to the variability and insufficient means for coping with the variations.
3. Lost revenues due to shortages, which have been caused by the variations.
4. The productivity of invested capital in operations becomes substandard as revenues are lost.
5. Decision-makers react to the fluctuations in demand and make investment decisions or change capacity plans to meet peak demands. These decisions are probably misguided, as peak demands may be eliminated by reorganisations of the supply chain.
6. Demand variations cause variations in the logistics chain, which again cause fluctuations in the planned use of transportation capacity. This will again produce sub-optimal transportation schemes and increase transportation costs.
7. Demand fluctuations caused by the bullwhip effect may cause missed production schedules, which actually are completely unnecessary, as there are no real changes in the demand, only inefficiencies in the supply chain.

There are some studies [5], which have revealed four key reasons for the occurrence of the bullwhip effect. These include (i) the updating of demand forecasts, (ii) order batching, (iii) price fluctuations and (iv) rationing and shortage gaming.

The updating of demand forecasts appears to be a major source of the bullwhip effect.

The parties of the supply chain build their forecasts on the historical demand patterns of their immediate customers. In this way, only the retailers build on the actual demand patterns of the customers, the other parties adjust to (unmotivated) fluctuations in the ordering policies of those preceding them in the supply chain. Another effect will also occur: if everybody reacts to fluctuations with smoothing techniques (like exponential smoothing), the fluctuations will amplify up through the supply chain. It appears that safety stocks, which are popular smoothing devices, will actually amplify the bullwhip effect.

The *order batching* will appear in two different forms: (i) periodic ordering and (ii) push ordering. In the first case there are a number reasons for building batches of individual orders. The costs for frequent order processing may be high, which will force customers into periodic ordering; this will in most cases destroy customer demand patterns. There are material requirement planning systems in use, which are run periodically and thus will cause that orders are placed periodically. Logistics operators often favor FTL-batches and will determine their tariffs accordingly. These reasons for periodic ordering are quite rational, and will, when acted upon, amplify variability and contribute to the bullwhip effect. Push ordering occurs, as the sales people employed by the producers try to meet their end-of-quarter or end-of-year bonus plans.

The effect of this is to amplify the variability with orders from customers overlapping end-of-quarter and beginning-of-quarter months, to destroy connections with the actual demand patterns of customers and to contribute to the bullwhip effect.

The producers initiate and control the price fluctuations for various reasons. Customers are driven to buy in larger quantities by attractive offers on quantity discounts, price discounts, coupons or rebates. Their behavior is quite rational: to make the optimal use of opportunities when prices shift between high and low. The problem introduced by this behavior is that buying patterns will not reflect consumption patterns anymore, customers buy in quantities which do not reflect their needs. This will amplify the bullwhip effect. The consequences are that producers (rightfully) suffer: manufacturing is on overtime during campaigns, premium transportation rates are paid during peak seasons and products suffer damages in overflowing storage spaces.

The rationing and shortage gaming occurs when demand exceeds supply. If the producers once have met shortages with a rationing of customer deliveries, the customers will start to exaggerate their real needs when there is a fear that supply will not cover demand. The shortage of DRAM chips and the following strong fluctuations in demand was a historic case of the rationing and shortage game. The bullwhip effect will amplify even further if customers are allowed to cancel orders when their real demand is satisfied. The gaming leaves little information on real demand and will confuse the demand patterns of customers.

It is a fact that these four causes of the bullwhip effect may be hard to monitor, and even harder to control in the forest products industry. We should also be aware of the fact that the four causes may interact, and act in concert, and that the resulting combined effects are not clearly understood, neither in theory nor in practice. It is also probably the case that the four causes are dependent on the supply chain's infrastructure and on the strategies used by the various actors.

The factors driving the bullwhip effect appear to form a hyper-complex, i.e. a system where factors show complex interactive patterns. The theoretical challenges posed by a hyper-complex merit study, even if significant economic consequences would not have been involved. The costs incurred by the consequences of the bullwhip effect offer a few more reasons for carrying out serious work on the mechanisms driving the bullwhip. Thus, we have built a theory to explain at least some of the factors and their interactions, and we have created a support system to come to terms with them and to find effective means to either reduce or eliminate the bullwhip effect.

With a little simplification there appears to be three possible approaches to counteract the bullwhip effect:

1. Find some means to share information from downstream of the supply chain with all the preceding actors.

2. Build channel alignment with the help of some co-ordination of pricing, transportation, inventory planning and ownership - when this is not made illegal by anti-trust legislation.

3. Improve operational efficiency by reducing cost and by improving on lead times.

The first approach can probably be focused on finding some good technology to accomplish the information sharing, as this can be shown to be beneficial for all the actors operating in the supply chain.

The second approach can first be focused on some non-controversial element, such as the co-ordination of transportation or inventory planning, and then the alignment can be widened to explore possible interactions with other elements.

The third approach is probably straight-forward: find inefficiencies in operations in selected strategic business units (SBUs), find ways to reduce costs and to improve on lead times, and explore if these solutions can be generalised for more actors in the supply chain.

The most effective - and the most challenging - effort will be to find ways to combine elements of all three approaches and to find synergistic programs, which will have the added benefit of being very resource-effective, to eliminate the bullwhip effect.

2. The Bullwhip Effect in the Forest Products Industry

The two corporate members of the EM-S Bullwhip consortium had observed the bullwhip effects in their own markets and in their own supply chains for fine paper products.

They also readily agreed that the bullwhip effect is causing problems and significant costs, and that any good theory or model, which could give some insight into dealing with the bullwhip effect, would be a worthwhile effort in terms of both time and resources.

There are several reasons why the bullwhip effect occurs in the fine paper products market. The first reason is to be found in the structure of the market (cf. fig. 1):

The paper mills do not deal directly with their end-customers, but fine paper products are distributed through wholesalers, merchants and retailers. The paper producers may Silvaculture & Timber Farming Logging & Chipping Pulp Manufacturing Paper Manufacturing Converting Operations Merchanting &

Distribution End-Users [Printing houses, etc.] Figure 1: The supply chain of the market for fine paper products.

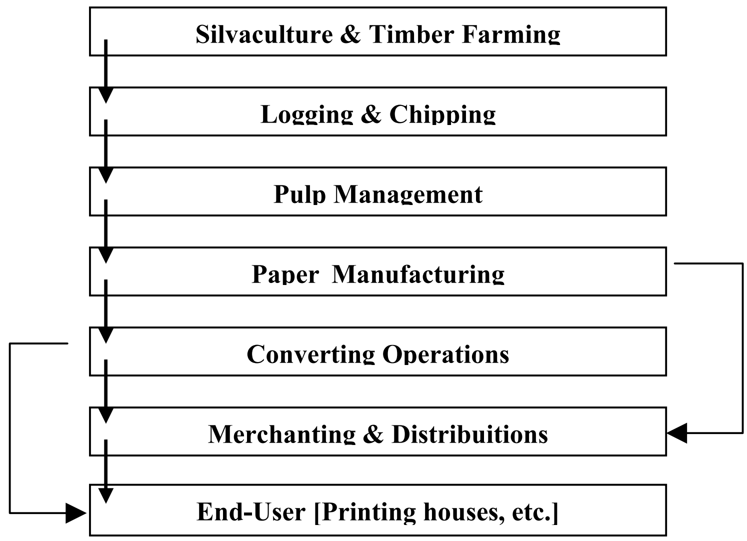

Figure 1: The supply chain of the market for fine paper products.

(i) own some of the operators in the market supply chain, (ii) they may share some of them with competitors or (iii) the operators may be completely independent and bound to play the market game with the paper producers. The operators in the market supply chain do not willingly share their customer and market data, information and knowledge with the paper producers.

Thus, the paper producers do not get *neither precise nor updated information* on the real customer demand, but get it in a filtered and/or manipulated way from the market supply chain operators. Market data is collected and summarised by independent data providers, and market forecasts are produced by professional forest products consultants and market study agencies, but still it appears that these macro level studies and forecasts do not exactly apply for the markets of a single paper producer. The market information needed for individual operations still needs to come from the individual market. Here the operators of the market supply chain control access to the data sources, and the paper producer is forced to build his demand forecasts on the numbers he get from the wholesaler or merchant part of the market supply chain.

The second reason for the bullwhip effect to occur is found earlier in the supply chain.

The demand and price fluctuations of the pulp markets dominate also the demand and price patterns of the paper products markets, even to such an extent, that the customers for paper products anticipate the expectations on changes in the pulp markets and act accordingly. If pulp prices decline, or are expected to decline, demand for paper products will decline, or stop in anticipation of price reductions. Then, eventually, prices will in fact go down as the demand has disappeared and the paper producers get nervous. The initial reason for fluctuations in the pulp market may be purely speculative, or may have no reason at all. Thus, the construction of any reasonable, explanatory cause-effect relationships to find out the market mechanisms that drive the bullwhip may be futile. If we want to draw an even more complex picture we could include the interplay of the operators in the market supply chain: their anticipations of the reactions of the other operators and their individual, rational (possibly even optimal) strategies to decide how to operate. This is a later task, to work out a composite bullwhip effect among the market supply chain operators, as we cannot deal with this more complex aspect here.

The third reason for the bullwhip effect is order batching. The logistics systems favour the shipping of larger batches of paper products, the building of inventories in the supply chain to meet demand fluctuations and push ordering to meet end-of-quarter or end-of-year financial needs. The logistics operators are quite often independent of both the paper producers and the wholesalers and/or retailers, which will make them want to operate in such a way that their result and financial goals are met. Thus they decide their own tariffs in such a way that their operations are effective and profitable, which will - in turn - affect the decisions of the market supply chain operators, including the paper producers. The adjustment to proper shipload or FTL batches will drive the bullwhip effect.

There is a fourth reason for the bullwhip effect, which is caused by the paper producers themselves. There are attempts at influencing or controlling the paper products markets by having occasional low price campaigns or special offers. The market supply chain operators react by speculating in the timing and the level of low price offers and will use the (rational) policy of buying only at low prices for a while. The nervous reactions and speculations of all the players drive the bullwhip effect.

There is a fifth and final explanation for the bullwhip effect, the so-called rationing and shortage game. This would be the case when demand in paper products markets outgrow the production capacity and the supply chain operators cannot fill their orders.

If they get less than their needs in some period, they will try to compensate for this in their next order by anticipating the rationing and ordering more than their actual needs. If the policy allows for not taking delivery of excessive orders, or if there is sufficient inventory capacity, the rationing game will drive the bullwhip effect. It is questionable if this factor will play any role in the paper products market, as the rule for the last decade seems to have been that there is more production capacity available than needed.

The bullwhip effect may be illustrated as in fig.2. The variations shown in fig.2 are simplifications, but the following patterns appear: (i) the printer (an end-customer) orders once per quarter according to the real market demand his has or is estimating; (ii) the dealer meets this demand and anticipates that the printer may need more (or less) than he orders; the dealer acts somewhat later than his customer; (iii) the paper mill reacts to the dealer's orders in the same fashion and somewhat later than the dealer. The resulting overall effect is the bullwhip effect.

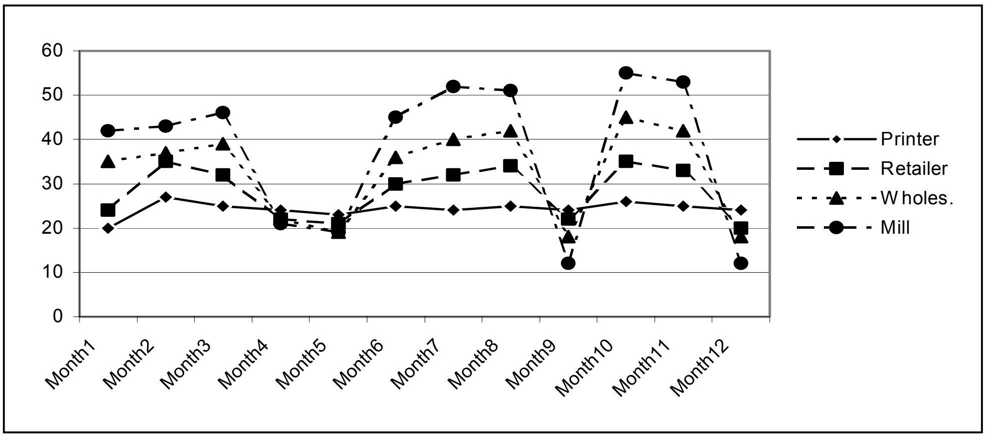

Figure 2: The bullwhip effect in the paper products market.

In the following section, we will present the standard theory for explaining the bullwhip and for coming to terms with it.

3. Explanations for the bullwhip effect: Standard Results

The first detailed, theoretical discussion of the bullwhip effect was published by Lee et al [5] in Management Science (April 1997) and in a more popular, empirical version in the Sloan Management Review [6] (Spring 1997). These two papers were continuations of earlier studies of the benefits of co-ordination among members of a supply chain, which comprises manufacturers, distributors, wholesalers and retailers.

A key mechanism for the co-ordination is the information flows among members of the supply chain, as they have a direct impact on production scheduling, inventory control, delivery planning and logistics solutions among the individual members of the supply chain. If this information flow gets distorted or interrupted the consequences will be felt in the production plans and the production scheduling, which will cause inventories to either grow rapidly or empty out, which again disrupts delivery planning and causes expensive logistics solutions. The dynamics behind this series of events - if mastered - will have a profound effect on how the supply chain is managed and will point the way to more productive and effective operations in all parts of the supply chain.

The supply chain management is an important issue for the Finnish forest products industry as supply chain mismanagement, with a total turnover of more than 100 BFIM, has consequences costing 100-200 MFIM per year.

Lee et al focus their study on the demand information flow and work out a theoretical framework for studying the effects of systematic information distortion as information works its way through the supply chain. The distortion becomes visible when the retailer's orders to the wholesaler do not coincide with the actual retail sales. As the wholesaler reacts to his retailers orders by adding a similar "safety margin" (a positive margin for growing demand, a negative margin for declining demand) when ordering from the producer, the distortion grows in magnitude.

This means that the orders to a supplier will have larger variance than the sales to a buyer. The theory demonstrates and proves that the distortion propagates upstream in an amplified form, i.e. the variances grow as we move upstream in the supply chain. This phenomenon is known as the bullwhip or whiplash effect.

Lee et al simplifies the context for the theoretical work by defining an idealised situation. They start with a multiple period inventory system, which is operated under a periodic review policy. They include the following assumptions: (i) past demands are not used for forecasting, (ii) re-supply is infinite with a fixed lead time, (iii) there is no fixed order cost, and (iv) purchase cost of the product is stationary over time. If the demand is stationary, the standard optimal result for this type of inventory system is to order up to S, where S is a constant. The optimal order quantity in each period is exactly equal to the demand of the previous period, which means that orders and demand have the same variance (and there is no bullwhip effect).

This idealised situation is useful as a starting point, as is gives a good basis for working out the consequences of distortion of information in terms of the variance, which is the indicator of the bullwhip effect. By relaxing the assumptions (i)-(iv), one at a time, it is possible to produce the bullwhip effect.

3.1 Demand Signal Processing

Lets focus on the retailer-wholesaler relationship (the framework applies also to a wholesaler-distributor or distributor-producer relationship). Now we consider a multiple period inventory model where demand is non-stationary over time and demand forecasts are updated from observed demand.

Lets assume that the retailer gets a much higher demand in one period. This will be interpreted as a signal for higher demand in the future, the demand forecasts for future periods get adjusted, and the retailer reacts by placing a larger order with the wholesaler. As the demand is non-stationary, the optimal policy of ordering up to S also gets non-stationary. A further consequence is that the variance of the orders grows, which is starting the bullwhip effect. If the lead-time between ordering point and the point of delivery is long, uncertainty increases and the retailer adds a "safety margin" to S, which will further increase the variance and add to the bullwhip effect.

Lee et al simplifies the context even further by focusing on a single-item, multiple period inventory, in order to be able to work out the exact bullwhip model.

The timing of the events is as follows: At the beginning of period t, a decision to order a quantity z_t is made. This time point is called the "decision point" for period t. Next the goods ordered v periods ago arrive. Lastly, demand is realized, and the available inventory is used to meet the demand. Excess demand is backlogged. Let S_t denote the amount in stock plus on order (including those in transit) after decision z_t has been made for period t. Lee at al assume that the retailer faces serially correlated demands which follow the process

$$D_t = d + \rho D_{t-1} + u_t$$

where D_t is the demand in period t, ρ is a constant satisfying $-1 < \rho < 1$, and u_t is independent and identically normally distributed with zero mean and variance σ^2. Here σ^2 is assumed to be significantly smaller than d, so that the probability of a negative demand is very small. The existence of d, which is some constant, basic demand, is doubtful; in the forest products markets a producer cannot expect to have any "granted demand". The use of d is technical, to avoid negative demand which will destroy the model, and it does not appear in the optimal order quantity. After formulating the cost minimization problem Lee et al. proved the following theorem,

Theorem 3.1. [5] *In the above setting, we have:*

1. *If $0 < \rho < 1$, the variance of retails orders is strictly larger than that of retail sales; that is,*

$$\mathrm{Var}(z_1) > \mathrm{Var}(D_0)$$

2. *If $0 < \rho < 1$, the larger the replenishment lead time, the larger the variance of orders; i.e. $Var(z_1)$ is strictly increasing in v.*

This theorem has been proved using the relationships

$$z_1^* = S_1 - S_0 + D_0 = \frac{\rho(1-\rho^{v+1})}{1-\rho}(D_0 - D_{-1}) + D_0 \qquad (1)$$

and

$$Var(z_1^*) = Var(D_0) + \frac{2\rho(1-\rho^{v+1})(1-\rho^{v+2})}{(1+\rho)(1-\rho)^2} \succ Var(D_0)$$

where z_1^* denotes the optimal amount of order. Which collapses into

$$Var(z_1^*) = Var(D_0) + 2\rho$$

for $v = 0$.

The optimal order quantity is an optimal ordering policy, which sheds some new light on the bullwhip effect. The effect gets started by rational decision making, i.e. by decision makers doing the best they can. In other words, there is no hope to avoid the bullwhip effect by changing the ordering policy, as it is difficult to motivate people to act in an irrational way. Other means will be necessary (cf. Section 5).

3.2 The Rationing Game

For the second reason for the bullwhip effect, we have to change the context. Now we have the situation that the demand potentially exceeds supply due to limitations of production capacity, uncertainty of production yield, or limitations or disturbances of the logistics system. We will focus on limitations of production capacity, we will work with a single-product, one-period inventory model, and we will assume multiple retailers. This is also known as the newsvendor problem, after a classical inventory problem in operations research.

In the forest products industry limitations of production capacity, to the extent that demand exceeds production capacity, is rather rare. Thus the rationing game does not occur very often, but we have included the theory in order to make the theoretical framework complete.

Lets assume that a producer supplies a single product to n identical retailers. Retailer i observes the demand distribution $\Phi(\cdot)$, and places an order z_i at time 0. Here we have a demand, which is known through its probability distribution, and the retailer tries to anticipate what the demand is going to be. The producer delivers the order at time 1; the producer's delivery m is a random variable, distributed according to $F(\cdot)$.

In the newsvendor problem, in which old newspapers are worthless and no backorders are possible, the optimal policy is to estimate the expected demand and its variance, and then to find an order quantity for which the marginal profit for adding one more newspaper to the order exactly balances the expected loss of leaving the order one newspaper short. Let us assume that this optimal order quantity is z'.

If the total amount of orders received from the n retailers exceeds the production quantity m, the producer will allocate products to the retailers in relation to their orders; if the total amount of orders is less than m, then all retailers will get the amounts they ordered. The question now is what the optimal order quantity z_i of retailer *i* should be?

The model gets rather complicated as it turns out that the retailer should not only try to estimate the expected demand and its variance, and then use the newsvendor solution, but should also in ordering try to anticipate what the other n - 1 retailers will do. The policy would be to order more than the expected demand in anticipation of rationing in order to make sure that the actual delivery will satisfy the expected demand. The only problematic issue is to decide how much to exaggerate and also to find out if the coming period will be one in which the production capacity will not b enough to satisfy all retailers. The model to be used for this is the classical Nash equilibrium model, which shows how to decide the optimal ordering policy (minimising expected cost = holding cost + ordering cost + shortage cost) when n - 1 other rational decision makers optimal decisions are given. There is an optimal ordering policy for retailer *i*, which is found at an equilibrium point - the so-called Nash equilibrium - which ensures that none of the retailers should try to exaggerate more than any other retailer.

The result is that the $z_i^* \leq z'$, and that the inequality hold strictly if $F(\cdot)$ and $\Phi(\cdot)$ are strictly increasing. The retailer should thus exaggerate the order to the producer and order more than the expected demand. If the demand is stationary, or slowly increasing, and the production capacity is expanded in time with the demand, then the optimal ordering policy will be very close to the newsvendor solution. The same holds if the number of retailers decreases so that total demand is in balance with the production capacity. In all other cases variance will increase and start the bullwhip effect.

The rationing game can play out over several layers in the supply chain. Consider a supply chain of three layers: a producer, multiple wholesalers and multiple retailers.

If the producer announces, or appears to have, shortages in his production capacity the wholesalers will play the rationing game in order to get enough delivery to satisfy their retailers. The retailers find out about the problems and will assume that there may be shortages, and will play the rationing game with their respective wholesaler.

Thus, demand and its variance are amplified as we move up the supply chain. Again all decisions can be proved rational, and even optimal, and the variance gets amplified and drives the bullwhip effect.

It can be shown (probably not easily) that if each retailer and wholesaler works with the demand signal processing models, and then combines them with the rationing games, the resulting overall bullwhip effect will be enhanced with an order of magnitude. This should be remembered when a fine paper producer is negotiating with distributors, wholesalers and retailers about implementing ordering and logistics solutions.

3.3 Order Batching

Now we need to change the context once more. Consider an inventory system with periodic reviews and full backlogging at a retailer. Let us assume that the demand is stationary, in which case the optimal order policy is to order up to S, which is equal to the previous review cycle's demand in every review cycle. Let us further assume that we have n retailers, all of which use a periodic review system with a review cycle of r periods, and that each retailer faces a demand pattern with mean m and variance σ^2.

Let us then focus on the wholesaler. From his perspective he has *n* retailers acting together through (i) random ordering, (ii) (positively) correlated ordering and (iii) balanced ordering, and it is evident that the use of his production capacity will be affected by the ordering patterns of the retailers. If all the retailers need to get their deliveries at exactly the same time, his production will run differently as compared to deliveries taking place uniformly in the order/production/delivery cycle.

Lee et al shows that with random ordering, each retailer appears with an order randomly during the cycle, the demand variance as seen by the wholesaler is the same as the demand variance seen by the retailer, if the review cycle r = 1. If the review cycle is longer, the wholesaler's demand variance will always be larger and the bullwhip effect gets initiated.

In the case of (positively) correlated ordering, we have an extreme situation, that all retailers order at exactly the same instance of the review cycle r. Lee et al shows that the resulting variance is much larger than the variance for random ordering. This is 2 quite understandable, as the wholesaler will have ordering peeks on (for instance) one day and nothing during the rest of the review cycle. If ordering and delivery policies are negotiated with retailers, a wholesaler should take care to avoid a situation where the retailers find it beneficial to use positively correlated ordering, as it for sure will drive the bullwhip effect.

The ideal case is one in which the retailers order in a way, which is evenly distributed in the review cycle r. In order for this to happen, the wholesaler needs a co-ordination scheme in which the retailers are organised in groups. Then all retailers in the same group order in a designated period within the review cycle r, and no other group orders in the same period. Lee et al shows that this scheme gives the wholesaler the smallest variance. This is reasonable, because the wholesaler can use his resources evenly during the review cycle and can estimate the order quantities to come in from each group.

Thus, different ordering patterns generate different variances in the review period. *Correlated ordering* with all orders appearing at the same instance shows the largest variance. *Balanced ordering* with all orders perfectly co-ordinated shows the lowest variance, and the *random ordering* falls in between the two other patterns. In all cases Lee et al was able to prove, that the variance experienced by the wholesaler was larger than the variance of any chosen retailer, which shows that the bullwhip effect is present with all three ordering patterns.

The ordering pattern models can be extended to a three-layer supply chain: retailer, wholesaler and producer, in which case we get combinations of ordering patterns as the wholesaler would not necessarily use the same ordering pattern as his retailers. The variance the producer experiences will in most cases be larger than the variance seen by the wholesaler. Rational decision making will force the wholesaler to replicate and amplify the variance he is getting from the retailers with his producer.

There is an ideal case, in which the bullwhip effect can be eliminated. If the wholesaler can get his retailers to form one single group for the review cycle, and then can persuade them to agree on their total orders *m* (as well as agreeing among themselves. how to share this total amount) then there will be no bullwhip effect. Such a case is a bit hard to find in the forest products markets, as the EU would not look kindly on attempts of this kind to limit competition.

3.4 Price Variations

Let us change context one final time. We will assume that a retailer faces an independent demand $\Phi(\cdot)$ for each period (but identically distributed for all periods). The wholesaler, the only source for the retailer, alternates between two prices c^L and c^H over time. The retailer perceives that the alternating is random with probabilities q and $(1 - q)$ for c^L and c^H respectively.

Lee et al shows that the optimal ordering policy for the retailer is to determine two ordering levels, S^L for c^L and S^H for c^H . Then, as the price is low order as much as 3 possible (i.e. S^L) and as the price is high order as little as possible (i.e. S^L) or nothing at all. It is clear that this will drive the bullwhip effect, as it is an optimal policy for the retailer to allow the orders to fluctuate with the price variations. It can be shown that even anticipated price variations will introduce the bullwhip effect as it is an optimal policy to adapt orders to anticipated price variations.

If we extend the price variations to more layers of the supply chain and include the producer, it is clear that the price variations reflected by the wholesaler have their origins with the producer. It would be an extreme, speculative case if the wholesaler could generate price variations on his own. Thus, the producer will face an optimal ordering policy from the wholesaler and find out that he gets orders only when he is offering the low price c^L. The variance faced by the wholesaler gets amplified as it is passed to the producer and the bullwhip effect seen by the producer is much stronger.

Inversely, it is possible to argue that if the producer refrains from price variations, and declares this policy publicly, then the bullwhip effect could be significantly reduced as the retailers would not allow the wholesalers to introduce any significant price variations. In the forest products market this would be possible but for the price variations forced by the strongly varying pulp prices, which seem to follow a logic of their own and which appears to be very difficult to forecast.

4. Some properties of fuzzy numbers

A fuzzy number A is a fuzzy set of the real line with a normal, (fuzzy) convex and continuous membership function of bounded support. The family of fuzzy numbers will be denoted by *F*. A $\gamma-$level set of a fuzzy number A is defined by $[A]^\gamma = \{t \in \Re | A(t) \geq \gamma\} if\ \gamma > 0$ and $[A]^\gamma = cl\{t \in \Re | A(t) > 0\}$ (the closure of the support of A) if $\gamma = 0$. If $A \in F$ a fuzzy number then $[A]^\gamma$ is a closed convex (compact) subset of $\Re$ for all $\gamma \in [0,1]$ and Let us introduce the notations

$$a_1(\gamma) = \min[A]^\gamma, \quad a_2(\gamma) = \max[A]^\gamma$$

In other words, $a_1(\gamma)$ denotes the left-hand side and $a_2(\gamma)$ denotes the right-hand side of the $\gamma-$cut. We shall use the notation

$$[A]^\gamma = [a_1(\gamma), a_2(\gamma)].$$

The support of A is the open interval $(a_1(0), a_2(0))$.

Let $[A]^\gamma = [a_1(\gamma),\, a_2(\gamma)]$ and $[B]^\gamma = [b_1(\gamma),\, b_2(\gamma)]$. be fuzzy numbers and let $\lambda \in \Re$ be a real number. Using the extension principle we can verify the following rules for addition and scalar multiplication of fuzzy numbers

$$[A+B]^\gamma = [a_1(\gamma)+b_1(\gamma), a_2(\gamma)+b_2(\gamma)], \quad [\lambda A]^\gamma = \lambda [A]^\gamma$$

Definition 4.1. *A fuzzy set A is called triangular fuzzy number with peak (or center) a, left width* $\alpha > 0$ *and right width* $\beta > 0$ *if its membership function has the following form*

$$A(t) = \begin{cases} 1 - \dfrac{a-t}{\alpha} & \text{if } a-\alpha \le t \le a \\ 1 - \dfrac{t-a}{\beta} & \text{if } a \le t \le a+\beta \\ 0 & \text{otherwise} \end{cases}$$

and we use the notation $A = (a, \alpha, \beta)$. *It can easily be verified that*

$$[A]^\gamma = [a-(1-\gamma)\alpha, a+(1-\gamma)\beta], \; \forall \gamma \in [0,1].$$

A triangular fuzzy number with center *a* may be seen as a fuzzy quantity "x is approximately equal to *a*".

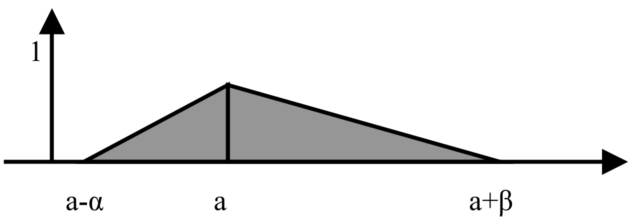

Figure 3: Triangular fuzzy number.

Let A and B be fuzzy numbers with $[A]^\gamma = [a_1(\gamma), a_2(\gamma)]$ and $[B]^\gamma = [b_1(\gamma), b_2(\gamma)]$.

We metricize the set of fuzzy numbers by the Hausdorff-metric

$$H(A,B)=\sup_{\gamma\in[0,1]}\max\{|a_1(\gamma)-b_1(\gamma)|,|a_2(\gamma)-b_2(\gamma)|\}.$$

i.e. H(A,B) is the maximal distance between the α-level sets of A and B. For example, if $A=(a,\alpha)$ and $B=(b,\alpha)$ are fuzzy numbers of symmetric triangular form with the same width $\alpha>0$ then H(A,B) = |a - b|, and if $A=(a,\alpha)$ and $B=(b,\beta)$ then

$$H(A,B)=|a-b|+|\alpha-\beta|.$$

Let $A\in F$ be a fuzzy number with $[A]^\gamma=[a_1(\gamma),a_2(\gamma)]$, $\gamma\in[0,1]$. We shall use the Goetschel-Voxman defuzzification method [4] to define E(A), the mean (or expected) value of A by

$$E(A)=\frac{\int_0^1 \gamma\cdot\frac{a_1(\gamma)+a_2(\gamma)}{2}d\gamma}{\int_0^1 \gamma\, d\gamma}=\int_0^1 \gamma(a_1(\gamma)+a_2(\gamma))d\gamma,$$

i.e. the weight of the arithmetic mean of $a_1(\gamma)$ *and* $a_2(\gamma)$ is just γ. It is easy to see that if $A=(a,\alpha,\beta)$ is a triangular fuzzy number then

$$E(A)=\int_0^1 \gamma[a-(1-\gamma)\alpha+a+(1-\gamma)\beta]d\gamma=a+\frac{\beta-\alpha}{6}.$$

Specially, when A = (a, α) is a symmetric triangular fuzzy number we get E(A) = a.

It can be shown [1] that E: $F\to\Re$ is a linear function (with respect to addition and multiplication by scalar of fuzzy numbers).

Theorem 4.1. *Let* $[A]^\gamma=[a_1(\gamma),a_2(\gamma)]$ *and* $[B]^\gamma=[b_1(\gamma),b_2(\gamma)]$ *be fuzzy numbers and let* $\lambda\in R$ *be a real number. Then*

$$E(A+B)=E(A)+E(B), \quad E(\lambda A)=\lambda E(A),$$

where the addition and multiplication by a scalar of fuzzy numbers is defined by the sup-min extension principle [9].

In [1] we introduced the (possibilistic) variance of $A \in F$ as

$$\begin{aligned} Var(A) &= \int_0^1 Pos[A \le a_1(\gamma)]\left(\left[\frac{a_1(\gamma)+a_2(\gamma)}{2}-a_1(\gamma)\right]^2\right)d\gamma \\ &\quad + \int_0^1 Pos[A \ge a_2(\gamma)]\left(\left[\frac{a_1(\gamma)+a_2(\gamma)}{2}-a_2(\gamma)\right]^2\right)d\gamma \\ &= \int_0^1 \gamma\left(\left[\frac{a_1(\gamma)+a_2(\gamma)}{2}-a_1(\gamma)\right]^2+\left[\frac{a_1(\gamma)+a_2(\gamma)}{2}-a_2(\gamma)\right]^2\right)d\gamma \\ &= \frac{1}{2}\int_0^1 \gamma(a_2(\gamma)-a_1(\gamma))^2\,d\gamma. \end{aligned}$$

The variance of A is defined as the expected value of the squared deviations between the arithmetic mean and the endpoints of its level sets, i.e. the lower possibility-weighted average of the squared distance between the left-hand endpoint and the arithmetic mean of the endpoints of its level sets plus the upper possibility-weighted average of the squared distance between the right-hand endpoint and the arithmetic mean of the end- points their of its level sets. The standard deviation of A is defined by

$$\sigma_A = \sqrt{Var(A)}.$$

For example, if $A = (a, \alpha, \beta)$ is a triangular fuzzy number then

$$Var(A)=\frac{1}{2}\int_0^1 \gamma(a+\beta(1-\gamma)-(a-\alpha(1-\gamma)))^2\,d\gamma=\frac{(\alpha+\beta)^2}{24}.$$

especially, if A = (a, α) is a symmetric triangular fuzzy number then

$$Var(A)=\frac{\alpha^2}{6}.$$

If A is the characteristic function of the crisp interval [a, b] then

$$Var(A) = \frac{1}{2}\int_0^1 \gamma (b-a)^2 \, d\gamma = \left(\frac{b-a}{2}\right)^2$$

that is,

$$\sigma_A = \frac{b-a}{2}, \qquad E(A) = \frac{a+b}{2}.$$

In probability theory, the corresponding result is: if the two possible outcomes of a probabilistic variable have equal probabilities then the expected value is their arithmetic mean and the standard deviation is the half of their distance.

The covariance between fuzzy numbers A and B is defined as

$$Cov(A,B) = \frac{1}{2}\int_0^1 \gamma (a_2(\gamma) - a_1(\gamma))(b_2(\gamma) - b_1(\gamma)) d\gamma$$

The covariance measures how much the endpoints of the γ-level sets of two fuzzy numbers move in tandem. Let A = *(a, α)* and B = *(b, β)* be symmetric triangular fuzzy numbers. Then

$$Cov(A,B) = \frac{\alpha\beta}{6}.$$

In [1] we showed that the variance of linear combinations of fuzzy numbers can be computed in the same manner as in probability theory.

Theorem 4.2. *Let* $\lambda, \mu \in \Re$ *and let A and B be fuzzy numbers. Then*

$$Var(\lambda A + \mu B) = \lambda^2 \, Var(A) + \mu^2 \, Var(B) + 2|\lambda\mu| Cov(A, B)$$

where the addition and multiplication by a scalar of fuzzy numbers is defined by the sup-min extension principle.

As a special case of Theorem 4.2 we get $Var(\lambda A) = \lambda^2 Var(A)$ for any $\lambda \in \Re$ and

$$Var(A+B) = Var(A) + Var(B) + 2Cov(A,B).$$

Another important question is the relationship between the subsethood and the variance of fuzzy numbers. One might expect that $A \subset B$ (that is *A(x)* $\leq$ *B(x)* for all $x \in \Re$) should imply the relationship Var(A) $\leq$ Var(B) because A is considered a "stronger restriction" than *B*. The following theorem [1] shows that subsethood does entail smaller variance.

Theorem 4.3. *Let A, B* $\in \mathcal{F}$ *with A* $\subset$ *B. Then Var(A)* $\leq$ *Var(B).*

5. A fuzzy approach to demand signal processing

Let us consider equation (1) with trapezoidal fuzzy numbers

$$z_1^* = S_1 - S_0 + D_0 = \frac{\rho\left(1-\rho^{v+1}\right)}{1-\rho}\left(D_0 - D_{-1}\right) + D_0. \qquad (2)$$

Then from Theorem 4.2 we get

$$Var(z_1^*) = \left[\frac{\rho\left(1-\rho^{v+1}\right)}{1-\rho}\right]^2 Var\left(D_0 - D_1\right) + Var\left(D_0\right)$$
$$+ 2\left|\frac{\rho\left(1-\rho^{v+1}\right)}{1-\rho}\right| Cov\left(D_0 - D_{-1}, D_0\right) > Var\left(D_0\right).$$

so the simple adaptation of the probabilistic model (i.e. the replacement of probabilistic distributions by possibilistic ones) does not reduce the bullwhip effect.

We will show, however that by including better and better estimates of future sales in period one, D_1 , we can reduce the variance of z_1 by replacing the old rule for ordering (2) with an adjusted rule. Suppose now that a sequence of D_1^i, i = 1, 2, . . . can be derived such that

$$H\left(D_1, D_1^i\right) \leq H\left(D_1, D_1^j\right) \quad \text{if} \qquad i \geq j,$$

i.e. D_1^i is a better estimation of D_1 than D_1^j if $i \geq j$. We can reduce the variance of z_1 by replacing the old rule for ordering (2) with the adjusted rule

$$z_1^i = \left[\frac{\rho\left(1-\rho^{\nu+1}\right)}{1-\rho} \left(D_0 - D_{-1}\right) + D_0 \right] \cap D_1^i.$$

Really, from Theorem 4.3 and from

$$z_1^i = \left[\frac{\rho\left(1-\rho^{\nu+1}\right)}{1-\rho} \left(D_0 - D_{-1}\right) + D_0 \right] \cap D_1^i \subset z_1^*, \qquad (3)$$

we get

$$Var(z_1^i) < Var(z_1^*),$$

which means that the variance of the suggested optimal order, z_1^i, is getting smaller and smaller as D_1^i is getting sharper and sharper. The crisp value of the optimal order is defined that the most typical value of z_1^i, that is, its expected value, E(z_1^i).

It can be seen that, similarly to the probabilistic case, Var(z_1^i) is a strictly increasing function of the replenishment lead time ν. However if ν = 0 then equation (3) reads

$$z_1^i = \left[\rho\left(D_0 - D_{-1}\right) + D_0\right] \cap D_1^i$$

and, furthermore, if ρ tends to zero then

$$Var(z_1^i) \to Var(D_0 \cap D_1)$$

that is, the bullwhip effect can be completely eliminated.

6. A fuzzy logic controller to demand signal processing

If the participants of the supply chain do not share information, or they do not agree on the value of D_1 then we can apply a neural fuzzy system that uses an error correction learning procedure to predict z_1. This system should include historical data, and a supervisor who is in the position to derive some initial linguistical rules from past situations which would have reduced the bullwhip effect.

A typical fuzzy logic controller (FLC) describes the relationship between the change of the control $\Delta u(t) = u(t) - u(t-1)$ on the one hand, and the error *e(t)* (the difference between the desired and computed system output) and its change

$$\Delta e(t) = e(t) - e(t-1).$$

on the other hand. The actual output of the controller *u(t)* is obtained from the previous value of control *u(t-1)* that is updated by Δ*u(t)*. This type of controller was suggested originally by *Mamdani and Assilian* in 1975 and is called the *Mamdani-type* FLC. A prototypical rule-base of a simple FLC realising with three linguistic values {*negative, zero, positive*} is listed in the following

$\Re_1$: **If** *e* is "positive" and Δ*e* is "near zero" **then** Δ*u* is "positive"

$\Re_2$: **If** *e* is "negative" and Δ*e* is "near zero" **then** Δ*u* is "negative"

$\Re_3$: **If** *e* is "near zero" and Δ*e* is "near zero" **then** Δ*u* is "near zero"

$\Re_4$: **If** *e* is "near zero" and Δ*e* is "positive" **then** Δ*u* is "positive"

$\Re_5$: **If** *e* is "near zero" and Δ*e* is "negative" **then** Δ*u* is "negative"

Or in tabular form

Δe(t) | e(t) → ↓	N	ZE	P
N	N	ZE	P
ZE	N	ZE	P
P	ZE	P	P

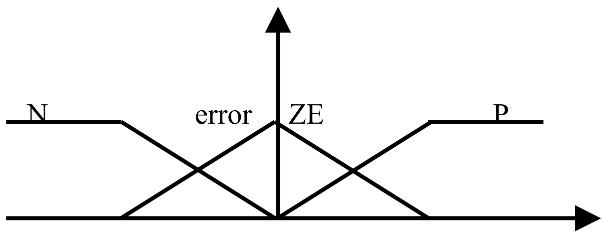

Figure 4: Initial membership functions for the *error*.

To easy the bullwhip effect we suggest the use of a fuzzy logic controller. Demand realizations D_{t-1} and D_{t-2} denote the volumes of retail sales in periods t - 1 and t - 2, respectively. We use a FLC to determine the change in *order*, denoted by Δz_1, in order to reduce the bullwhip effect, that is, the variance of z_1. We shall derive z_1 from D_0 , D_{-1} (sales data in the last two periods) and from the last order z_0 as

$$z_1 = z_0 + \Delta z_1$$

where the crisp value of Δz_1 is derived from the rule base $\{\Re_1, \dots, \Re_5\}$, where e = D_0 - z_0 is the difference between the past realized demand (sales), D_0 and order z_0, and the change of error

$$\Delta e := e - e_{-1} = (D_0 - z_0) - (D_{-1} - z_{-1})$$

is the change between (D_0 - z_0) and ($D_{-1} - z_{-1}$).

To improve the performance (approximation ability) we can include more historical data D_{t-3}, D_{t-4} … in the antecedent part of the rules. The problem is that the fuzzy system itself can not learn the membership function of Δz_1, so we should include a neural network to approximate the crisp value of z_1, which is the most typical value of $z_0 + \Delta z_1$.

It is here when the supervisor should provide crisp historical learning patterns for the concrete problem, for example,

{30, 5, 20}

which says us that if at some past situations (D_{k-1} - z_{k-1}) was 30 and (D_{k-2} - z_{k-2}) was 5 then the value of z_k should have been (z_{k-1} - 20) in order to reduce the bullwhip effect.

The meaning of this pattern can be interpreted as: if the preceding chain member ordered a little bit less than it sold in period (k-2) and much less in period (k-1) then its order for period k should have been enlarged by 20 in order to reduce the bullwhip effect (otherwise - at a later time - the order from this member would unexpectedly jump in order to meet his customers' demand - and that is the bullwhip phenomena).

Then the output of the neural fuzzy numbers is computed as the most typical value of the fuzzy system, and the system parameters (i.e. the shape functions of the error, change in error and change in order) are learned by the generalized δ learning rule (the error backpropagation algorithm).

7. Summary and Conclusions

The results reached in the EM-S Bullwhip project are mainly theoretical results, which also was the objective for the research program.

Nevertheless, many of the results we have found in working with both the standard Lee et al model and with the new EM-S Bullwhip model have practical implications. Even if we propose them as "ways to handle the bullwhip effect for paper mills", it should be clear that significant validation and verification work remains to be done before this statement will be fully true. The validation and verification process requires access to specific data on real market operations, preferably on the paper mill level.

As a tool for the verification and validation process, we built a prototype of a decision support system in which we have implemented causal models, which describe the four bullwhip-driving factors. The system is a platform for testing different policy solutions with the bullwhip models, for collecting data from different data sources with intelligent agents, which also update the models, and for running simulations of the processes involved with the PowerSim software package.

Nevertheless, quite some work is needed for re-engineering the logistics chain and getting its operators to agree on sharing information. This is essential, as we can hope to eliminate the bullwhip effect only to some degree if we have to keep estimating the activities of the supply chain operators.

If this negotiating process gets complex and time-consuming, a business unit can still work with the bullwhip effect internally by following up on its demand and sales pat- terns, and by trying to find ways to neutralise the most violent variations. For this purpose, the mathematical results we have found will be most useful. This work can be combined with running experiments with different solutions with the help of the PowerSim models.

References

[1] C. Carlsson and R. Fullér, On possibilistic mean value and variance of fuzzy numbers, *TUCS Technical Reports*, Turku Centre for Computer Science, No. 299/1999.

[2] C. Carlsson and R. Fullér, On possibilistic mean value and variance of fuzzy numbers*, Fuzzy Sets and Systems* (submitted).

[3] D. Dubois and H. Prade, The mean value of a fuzzy number, *Fuzzy Sets and Systems* 24 (1987) 279-300.

[4] R. Goetschel and W. Voxman, Elementary Fuzzy Calculus, *Fuzzy Sets and Systems,* 18 (1986) 31-43.

[5] H.L. Lee, V. Padmanabhan and S. Whang, Information distortion in a supply chain: The bullwhip effect, *Management Science*, 43(1997) 546-558.

[6] H.L. Lee, V. Padmanabhan and S. Whang, The Bullwhip Effect in Supply Chains, *Sloan Management Review*, Spring 1997 93-102.

[7] D. Heyman and M. Sobel, *Stochastic Models in Operations Research*, vol. I, McGraw-Hill, New York, 1984.

[8] J.A. Kahn, Inventories and the Volatility of Production, *American Economic Rev.*, 77(1987) 667-679.

[9] L.A. Zadeh, Fuzzy Sets, *Information and Control*, 8(1965) 338-353.