Project selection with fuzzy real options*

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Abstract

In this paper we shall represent strategic planning problems by dynamic decison trees, in which the nodes are projects that can be deferred or postponed for a certain period of time. Using the theory of real options we shall identify the optimal path of the tree, i.e. the path with the biggest real option value in the end of the planning period.

Keywords: Real option, decision tree, fuzzy number, dynamic decision tree.

1 Introduction

Many industries are experiencing changes that require large investments with substantial risk. Phasing and scheduling of projects which are related to each other can make a huge impact on the value of that set of projects. By phasing and scheduling projects, every step in a project opens or closes the possibility for further options. This is called a chain of growth options, or a compound growth option. Creating options can buy us time to think and gain information to decide whether or not go ahead with a certain bigger investment.

Decision trees are excellent tools for making financial decisions where a lot of vague information needs to be taken into account. They provide an effective structure in which alternative decisions and the implications of taking those decisions can be laid down and evaluated. They also help us to form an accurate, balanced picture of the risks and rewards that can result from a particular choice.

In this paper we shall represent strategic planning problems by dynamic decison trees, in which the nodes are projects that can be deferred or postponed for a certain

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period of time. Using the theory of real options we shall identify the optimal path of the tree, i.e. the path with the biggest real option value in the end of the planning period.

2 Real options

Real options is an important way of thinking about valuation and strategic decision-making, and the power of this approach is starting to change the economic ”equation” of many industries. Real options in option thinking are based on the same principals as financial options. In real options, the options involve ”real” assets as opposed to financial ones [1]. To have a ”real option” means to have the possibility for a certain period to either choose for or against something, without binding oneself up front. The value of a real option is computed by [7]

$$\mathrm{ROV} = S_0 e^{-\delta T} N(d_1) - X e^{-rT} N(d_2)$$

where

$$\begin{aligned} d_1 &= \frac{\ln(S_0/X) + (r - \delta + \sigma^2/2)T}{\sigma\sqrt{T}}, \\ d_2 &= d_1 - \sigma\sqrt{T}, \end{aligned} \tag{1}$$

and where S_0 is the present value of expected cash flows, $N(d)$ denotes the probability that a random draw from a standard normal distribution will be less than d, X is the (nominal) value of fixed costs, r is the annualized continuously compounded rate on a safe asset, T is the time to maturity of option (in years), σ is the uncertainty of expected cash flows, and finally δ is the value lost over the duration of the option.

The main question that a firm must answer for a deferrable investment opportunity is: *How long do we postpone the investment up to T time periods?* To answer this question, Benaroch and Kauffman ([2], page 204) suggested the following decision rule for optimal investment strategy:

Where the maximum deferral time is T, make the investment (exercise the option) at time t^*, $0 \le t^* \le T$, for which the option, C_{t^*}, is positive and attends its maximum value,

$$C_{t^*} = \max_{t=0,1,\ldots,T} C_t = V_t e^{-\delta t} N(d_1) - X e^{-rt} N(d_2), \tag{2}$$

where

$$\begin{aligned} V_t &= \mathrm{PV}(\mathrm{cf}_0, \ldots, \mathrm{cf}_T, \beta_P) - \mathrm{PV}(\mathrm{cf}_0, \ldots, \mathrm{cf}_t, \beta_P) \\ &= \mathrm{PV}(\mathrm{cf}_{t+1}, \ldots, \mathrm{cf}_T, \beta_P), \end{aligned}$$

that is,

$$V_t = \mathrm{cf}_0 + \sum_{j=1}^{T} \frac{\mathrm{cf}_j}{(1+\beta_P)^j} - \mathrm{cf}_0 - \sum_{j=1}^{t} \frac{\mathrm{cf}_j}{(1+\beta_P)^j}$$
$$= \sum_{j=t+1}^{T} \frac{\mathrm{cf}_j}{(1+\beta_P)^j},$$

and cf_t denotes the expected cash flow at time t, and β_P is the risk-adjusted discount rate (or required rate of return on the project, which is usually the project's beta).

Of course, this decision rule has to be reapplied every time new information arrives during the deferral period to see how the optimal investment strategy might change in light of the new information.

3 A fuzzy approach to real option valuation

Usually, the present value of expected cash flows can not be be characterized by a single number. We can, however, estimate the present value of expected cash flows by using a trapezoidal possibility distribution of the form

$$\tilde{S}_0 = (s_1, s_2, \alpha, \beta)$$

i.e. the most possible values of the present value of expected cash flows lie in the interval $[s_1, s_2]$ (which is the core of the trapezoidal fuzzy number S_0), and $(s_2 + \beta)$ is the upward potential and $(s_1 - \alpha)$ is the downward potential for the present value of expected cash flows.

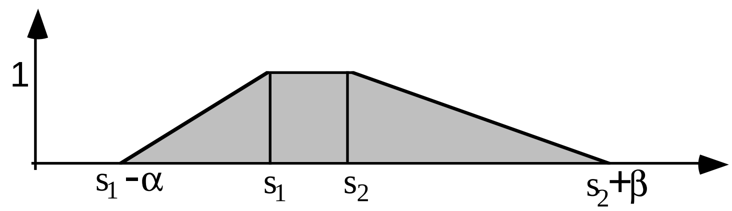

Figure 1: A possibility distribution of present values of expected cash flow.

In a similar manner we can estimate the expected costs by using a trapezoidal possibility distribution of the form

$$\tilde{X} = (x_1, x_2, \alpha', \beta'),$$

i.e. the most possible values of expected cost lie in the interval $[x_1, x_2]$ (which is the core of the trapezoidal fuzzy number X), and $(x_2 + \beta')$ is the upward potential and $(x_1 - \alpha')$ is the downward potential for expected costs.

In these circumstances we suggest the use of the following formula for computing fuzzy real option values

$$\tilde{C}_0 = \tilde{S}_0 e^{-\delta T} N(d_1) - \tilde{X} e^{-rT} N(d_2), \tag{3}$$

where,

$$d_1 = \frac{\ln(E(\tilde{S}_0)/E(\tilde{X})) + (r - \delta + \sigma^2/2)T}{\sigma\sqrt{T}},$$

$E(\tilde{S}_0)$ denotes the possibilistic mean value of the present value of expected cash flows, $E(\tilde{X})$ stands for the possibilistic mean value of expected costs and $\sigma := \sigma(\tilde{S}_0)$ is the possibilistic variance of the present value of expected cash flows [5].

In the following we shall generalize the probabilistic decision rule (2) for optimal investment strategy to fuzzy setting:

Where the maximum deferral time is T, make the investment (exercise the option) at time t^*, $0 \le t^* \le T$, for which the option, $\tilde{C}_{t^*}$, is positive and attends its maximum value,

$$\tilde{C}_{t^*} = \max_{t=0,1,\dots,T} \tilde{C}_t = \tilde{V}_t e^{-\delta t} N(d_1) - \tilde{X} e^{-rt} N(d_2), \tag{4}$$

where

$$\begin{aligned} \tilde{V}_t &= \mathrm{PV}(\tilde{\mathrm{cf}}_0, \dots, \tilde{\mathrm{cf}}_T, \beta_P) - \mathrm{PV}(\tilde{\mathrm{cf}}_0, \dots, \tilde{\mathrm{cf}}_t, \beta_P) \\ &= \mathrm{PV}(\tilde{\mathrm{cf}}_{t+1}, \dots, \tilde{\mathrm{cf}}_T, \beta_P), \end{aligned}$$

that is,

$$\begin{aligned} \tilde{V}_t &= \tilde{\mathrm{cf}}_0 + \sum_{j=1}^{T} \frac{\tilde{\mathrm{cf}}_j}{(1+\beta_P)^j} - \tilde{\mathrm{cf}}_0 - \sum_{j=1}^{t} \frac{\tilde{\mathrm{cf}}_j}{(1+\beta_P)^j} \\ &= \sum_{j=t+1}^{T} \frac{\tilde{\mathrm{cf}}_j}{(1+\beta_P)^j}, \end{aligned}$$

where $\tilde{\mathrm{cf}}_t$ denotes the expected (fuzzy) cash flow at time t, β_P is the risk-adjusted discount rate (or required rate of return on the project, which is usually the project's beta).

However, to find a maximizing element from the set

$$\{\tilde{C}_0, \tilde{C}_1, \dots, \tilde{C}_T\},$$

is not an easy task because it involves ranking of trapezoidal fuzzy numbers.

In our computerized implementation we have employed the following value function to order fuzzy real option values, $\tilde{C}_t = (c_t^L, c_t^R, \alpha_t, \beta_t)$, of trapezoidal form:

$$v(\tilde{C}_t) = \frac{c_t^L + c_t^R}{2} + r_A \cdot \frac{\beta_t - \alpha_t}{6},$$

where $r_A \geq 0$ denotes the degree of the investor's risk aversion. If $r_A = 0$ then the (risk neutral) investor compares trapezoidal fuzzy numbers by comparing their possibilistic expected values, i.e. he does not care about their downward and upward potentials.

4 Nordic Telekom Inc.

The World's telecommunications markets are undergoing a revolution. In the next few years mobile phones may become the World's most common means of communication, opening up new opportunities for systems and services. Nordic Telekom Inc. (NTI) is one of the most successful mobile communications operator in Europe[1] and has gained a reputation among its competitors as a leader in quality, innovations in wireless technology and customer relationships of long duration.

Still it does not have a dominating position in any of its customer segments, which is not even advisable in the European Common market, as there are always 4-8 competitors with sizeable market shares. NTI would, nevertheless, like to have a position which would be dominant against any chosen competitor when defined for all the markets in which NTI operates.

NPI has associated companies that provide GSM services in six countries: Finland, Norway, Sweden, Denmark, Estonia, Latvia and Lithuania.

We consider strategic decisions for the planning period 2002-2007. There are three possible alternatives for NPI: introduction of the third generation mobile solutions (3G), expanding its operation to other countries or develop new m-commerce solutions. The introduction of 3G system can be postponed by two years at max, the expansion may be delayed by a year at max and the project on introduction of new m-commerce solutions should start immediately.

Our goal is to maximize the company's cash flow in the end of the planning period (year 2007). In our computerized implementation we have represented NTI's strategic planning problem by a dynamic decision tree, in which the future expected cash flows and costs are estimated by trapezoidal fuzzy numbers. Then using the theory of fuzzy real options we have computed the real option values for

[1]NTI is a fictional corporation.

all nodes of the dynamic decision tree. Then we have selected the path with the biggest real option value in 2007.

5 Conclusions

Despite its appearance, the fuzzy real options model is quite practical and useful. Standard work in the field uses probability theory to account for the uncertainties involved in future cash flow estimates. This may be defended for financial options, for which we can assume the existence of an efficient market with numerous players and numerous stocks for trading, which may justify the assumption of the validity of the laws of large numbers and thus the use of probability theory. The situation for real options is quite different. The option to postpone an investment (which in our case is a very large - so-called giga - investment) will have consequences, differing from efficient markets, as the number of players producing the consequences is quite small.

The imprecision we encounter when judging or estimating future cash flows is not stochastic in nature, and the use of probability theory gives us a misleading level of precision and a notion that consequences somehow are repetitive. This is not the case, the uncertainty is genuine, i.e. we simply do not know the exact levels of future cash flows. Without introducing fuzzy real option models it would not be possible to formulate this genuine uncertainty. The proposed model that incorporates subjective judgments and statistical uncertainties may give investors a better understanding of the problem when making investment decisions.

References

[1] James Alleman and Eli Noam eds., *The New Investment Theory of Real Options and Its Implication for Telecommunications Economics*, Kluwer Academic Publishers, Boston 1999.

[2] M. Benaroch and R. J. Kauffman, Justifying electronic banking network expansion using real options analysis, *MIS Quarterly*, 24(2000) 197-225.

[3] P. Balasubramanian, N. Kulatikala and J. Storck, Managing information technology investments using a real-options approach, *Journal of Strategic Information Systems*, 9(2000) 39-62.

[4] F. Black and M. Scholes, The pricing of options and corporate liabilities, *Journal of Political Economy*, 81(1973) 637-659.

[5] C. Carlsson and R. Fullér, On possibilistic mean value and variance of fuzzy numbers, *Fuzzy Sets and Systems*, 122(2001) 315-326.

[6] D.Dubois and H.Prade, *Possibility Theory* (Plenum Press, New York,1988).

[7] K. J. Leslie and M. P. Michaels, The real power of real options, *The McKinsey Quarterly*, 3(1997) 5-22.

[8] Timothy A. Luehrman, Investment opportunities as real options: getting started on the numbers, *Harvard Business Review*, July-August 1998, 51-67.

[9] R. Merton, Theory of rational option pricing, *Bell Journal of Economics and Management Science*, 4(1973) 141-183.

[10] L.A. Zadeh, Fuzzy Sets, *Information and Control*, 8(1965) 338-353.