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TURKU CENTRE for COMPUTER SCIENCE

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#### Abstract

Certification of safety-critical systems requires formal verification of system properties and behaviour as well as quantitative demonstration of safety. Usually, formal modelling frameworks do not include quantitative assessment of safety. This has a negative impact on productivity and predictability of system development. In this paper we present an approach to integrating quantitative safety analysis into formal system modelling and verification in Event-B. The proposed approach is based on an extension of Event-B, which allows us to perform quantitative assessment of safety within proofbased verification of system behaviour. This enables development of systems that are not only correct but also safe by construction. The approach is demonstrated by a case study – an automatic railway crossing system.

**Keywords:** Event-B; formal modelling; refinement; safety; probabilistic reasoning

**TUCS Laboratory** Distributed Systems Laboratory

# 1 Introduction

Safety is a property of a system to not endanger human life or environment [4]. To guarantee safety, designers employ various rigorous techniques for formal modeling and verification. Such techniques facilitate formal reasoning about system correctness. In particular, they allow us to guarantee that a safety invariant – a logical representation of safety – is always preserved during system execution. However, real safety-critical systems, i.e., the systems whose components are susceptible to various kinds of faults, are not "absolutely" safe. In other words, certain combinations of failures might lead to an occurrence of a hazard – a potentially dangerous situation that breaches safety requirements. While designing and certifying safety-critical systems, we should demonstrate that the probability of a hazard occurrence is acceptably low. In this paper we propose an approach to combining formal system modeling and quantitative safety analysis.

Our approach is based on a probabilistic extension of Event-B [21]. Event-B is a formal modeling framework for developing systems correct-by-construction [3, 1]. It is actively used in the EU project Deploy [6] for modeling and verifying of complex systems from various domains including railways. The Rodin platform [20] provides the designers with an automated tool support that facilitates formal verification and makes Event-B relevant in an industrial setting.

The main development technique of Event-B is refinement – a top-down process of gradual unfolding of the system structure and elaborating on its functionality. In this paper we propose design strategies that allow the developers to structure safety requirements according to the system abstraction layers. Essentially, such an approach can be seen as a process of extracting a fault tree – a logical representation of a hazardous situation in terms of the primitives used at different layers of abstraction. Eventually, we arrive at the representation of a hazardous situation in terms of failures of basic system components. Since our specification contains an explicit representation of probabilities of component failures, standard calculations allow us to obtain a probabilistic evaluation of a hazard occurrence. As a result, we obtain an algebraic representation of probability of safety requirements violation. This probability is defined using the probabilities of system component failures. To illustrate our approach, we present a formal development and safety analysis of a radio-based railway crossing. We believe the proposed approach can potentially facilitate development, verification and assessment of safety-critical systems.

The rest of the paper is organised as follows. In Section 2 we describe our formal modelling framework – Event-B, and briefly introduce its probabilistic extension. In Section 3 we discuss a general design strategy for specifying Event-B models amenable for probabilistic analysis of system safety. In Sec-



Figure 1: Event-B machine and context

tion 4 we demonstrate the presented approach by a case study. Finally, Section 5 presents an overview of the related work and some concluding remarks.

# 2 Modelling in Event-B

The B Method [2] is an approach for the industrial development of highly dependable software. The method has been successfully used in the development of several complex real-life applications [19, 5]. Event-B is a formal framework derived from the B Method to model parallel, distributed and reactive systems. The Rodin platform provides automated tool support for modelling and verification (by theorem proving) in Event-B. Currently Event-B is used in the EU project Deploy to model several industrial systems from automotive, railway, space and business domains.

**Event-B Language.** In Event-B, a system specification (model) is defined using the notion of an *abstract state machine* [18]. An abstract state machine encapsulates the model state, represented as a collection of model variables, and defines operations on this state. Therefore, it describes the dynamic part (behaviour) of the modelled system. A machine may also have an accompanying component, called *context*, which contains the static part of the system. In particular, a context can include user-defined carrier sets, constants and their properties, which are given as a list of model axioms. A general form of Event-B models is given in Fig. 1.

The machine is uniquely identified by its name M. The state variables, v, are declared in the **Variables** clause and initialised in the *Init* event. The variables are strongly typed by the constraining predicates I given in the **Invariants** clause. The invariant clause also contains other predicates defining properties that must be preserved during system execution.

The dynamic behaviour of the system is defined by the set of atomic events specified in the **Events** clause. Generally, an event can be defined as

Action $(S)$	BA(S)
x := E(x, y)	$x' = E(x,y) \ \land \ y' = y$
$x:\in Set$	$\exists z  \cdot  (z \in Set \land x' = z) \land \ y' = y$
$x:\mid Q(x,y,x')$	$\exists z  \cdot  (Q(x,z,y) \wedge x' = z) \ \wedge \ y' = y$

Figure 2: Before-after predicates

follows:

#### $evt \cong any \ a \text{ where } g \text{ then } S \text{ end},$

where a is the list of local variables, the guard g is a conjunction of predicates over the local variables a and state variables v, while the action S is an assignment to the state variables. If an event does not have local variables, it can be described simply as

#### $evt \cong$ when g then S end.

The occurrence of events represents the observable behaviour of the system. The guard defines the conditions under which the action can be executed, i.e., when the event is *enabled*. If several events are enabled at the same time, any of them can be chosen for execution nondeterministically. If none of the events is enabled then the system deadlocks.

In general, the action of an event is a parallel composition of assignments. The assignments can be either deterministic or non-deterministic. A deterministic assignment, x := E(x, y), has the standard syntax and meaning. A nondeterministic assignment is denoted either as  $x :\in Set$ , where Set is a set of values, or x :| Q(x, y, x'), where Q is a predicate relating initial values of x, y to some final value of x'. As a result of such a non-deterministic assignment, x can get any value belonging to Set or satisfying Q.

**Event-B Semantics.** The semantics of Event-B actions is defined using so-called before-after (BA) predicates [3, 18]. A BA predicate describes a relationship between the system states before and after execution of an event, as shown in Fig. 2. Here x and y are disjoint lists (partitions) of state variables, and x', y' represent their values in the after-state.

The semantics of a whole Event-B model is formulated as a number of *proof obligations*, expressed in the form of logical sequents. The full list of proof obligations can be found in [3].

**Probabilistic Event-B.** In our previous work [21] we have have extended the Event-B modelling language with a new operator – *quantitative proba*-

*bilistic choice*, denoted  $\oplus$ . It has the following syntax

$$x \oplus | x_1 @ p_1; \ldots; x_n @ p_n,$$

where  $\sum_{i=1}^{n} p_i = 1$ . It assigns to the variable x a new value  $x_i$  with the corresponding non-zero probability  $p_i$ . The quantitative probabilistic choice (assignment) allows us to precisely represent the probabilistic information about how likely a particular choice is made. In other words, it behaves according to some known probabilistic distribution.

We have restricted the use of the new probabilistic choice operator by introducing it only to replace the existing demonic one. This approach has also been adopted by Hallerstede and Hoang, who have proposed extending the Event-B framework with *qualitative probabilistic choice* [10]. It has been shown that any probabilistic choice statement always refines its demonic nondeterministic counterpart [13]. Hence, such an extension is not interfering with the established refinement process. Therefore, we can rely on the Event-B proof obligations to guarantee functional correctness of a refinement step. Moreover, the probabilistic information introduced in new quantitative probabilistic choices can be used to stochastically evaluate certain non-functional system properties.

For instance, in [21] we have shown how the notion of Event-B refinement can be strengthened to quantitatively demonstrate that the refined system is more reliable than its abstract counterpart. In this paper we aim at enabling quantitative safety analysis within Event-B development.

# **3** Safety Analysis in Event-B

In this paper we focus on modelling of highly dynamic reactive control systems. Such systems provide instant control actions as a result of receiving stimuli from the controlled environment. Such a restriction prevents the system from executing automated error recovery, i.e. once a component fails, its failure is considered to be permanent and the system ceases its automatic functioning.

Generally, control systems are built in a layered fashion and reasoning about their behaviour is conducted by unfolding layers of abstraction. Deductive system safety analysis is performed in a similar way. We start by identifying a hazard – a dangerous undesirable situation associated with the system. By unfolding the layers of abstraction we formulate the hazard in terms of component states of different layers.

In an Event-B model, a hazard can be naturally defined as a predicate over the system variables. Sometimes, it is more convenient to reformulate a hazard as a dual safety requirement (property) that specifies a proper behaviour of a system in a hazardous situation. The general form of such a safety property is the following:

$$SAF \cong H(v) \Rightarrow K(v),$$

where the predicate H(v) specifies a hazardous situation and the predicate K(v) defines the safety requirements in the terms of the system variables and their states.

The essential properties of an Event-B model are usually formulated as invariants. However, to represent system behaviour realistically, our specification should include modelling of not only normal behaviour but also component failure occurrence. Since certain combinations of failures will lead to hazardous situations, we cannot guarantee "absolute" preservation of safety invariants. Indeed, the goal of development of safety-critical systems is to guarantee that the probability of violation of safety requirements is sufficiently small.

To assess the preservation of a desired safety property we will unfold it (in the refinement process) until it refers only to concrete system components that have direct impact on the system safety. To quantitatively evaluate this impact we require that these components are probabilistically modelled in Event-B using the available information about their reliability. Next we demonstrate how the process of unfolding the safety property from the abstract to the required concrete representation can be integrated into the system development by refinement in Event-B.

Often, functioning of a system can be structured according to a number of *execution stages*. There is a specific component functionality associated with each stage. Since there is no possibility to replace or repair failed system components, we can divide the process of quantitative safety assessment into several consecutive steps, where each step corresponds to a particular stage of the system functioning. Moreover, a relationship between different failures of components and the system behaviour at a certain execution stage is preserved during all the subsequent stages. On the other hand, different subsystems can communicate with each other, which leads to possible additional dependencies between system failures (not necessarily within the same execution stage). This fact significantly complicates quantitative evaluation of the system safety.

We can unfold system safety properties either in a *backward* or in a *forward* way. In the backward unfolding we start from the last execution stage preceding the stage associated with the potentially hazardous situation. In the forward one we start from the first execution stage of the system and continue until the last stage just before the hazardous situation occurs. In this paper we follow the former approach. The main idea is to perform a stepwise analysis of any possible behaviour of all the subsystems at every execution stage preceding the hazardous situation, while gradually unfolding the abstract safety property in terms of new (concrete) variables representing faulty components of the system.

Specifically, in each refinement step, we have to establish the relationship between the newly introduced variables and the abstract variables present in the safety property. A standard way to achieve this is to formulate the required relationship as a number of safety invariants in Event-B. According to our development strategy, each such invariant establishes a connection between abstract and more concrete variables that have an impact on system safety. Moreover, the preservation of a safety invariant is usually verified for a particular subsystem at a specific stage. Therefore, we can define a general form of such an invariant in the following way:

$$I_s(v, u) \cong F(v) \Rightarrow (K(v) \Leftrightarrow L(u)),$$

where the predicate F restricts the execution stage and the subsystems involved, while the predicate  $K \Leftrightarrow L$  relates the values of the newly introduced variables u with the values the abstract variables v present in the initially defined safety property or/and in the safety invariants defined in the previous refinement steps.

To calculate the probability of preservation of the safety property, the refinement process should be continued until all the abstract variables, used in the definition of the system safety property, are related to the concrete, probabilistically updated variables, representing various system failures or malfunctioning. The process of probability evaluation is rather straightforward and based on basic definitions and rules for calculating probabilities (see [7] for instance).

Let us consider a small yet generic example illustrating the calculation of probability using Event-B safety invariants. We assume that the safety property SAF is defined as above. In addition, let us define two safety invariants –  $I_s$  and  $J_s$  – introduced in two subsequent refinement steps. More specifically,

$$I_s \cong F \Rightarrow (K(v) \Leftrightarrow L_1(u_1) \lor L_2(u_2)) \text{ and } J_s \cong F \Rightarrow (L_2(u_2) \Leftrightarrow N(w)),$$

where  $u_1 \subset u, u_1 \neq \emptyset$  are updated probabilistically in the first refinement, while  $u_2 = u \setminus u_1$  are still abstract in the first refinement machine and related by  $J_s$  to the probabilistically updated variables w in the following one. Let us note that the predicate  $\tilde{F}$  must define the earlier stage of the system than the predicate F does. Then the probability that the safety property SAF is preserved by the system is

$$P_{SAF} = P\{K(v)\} = P\{L_1(u_1) \lor L_2(u_2)\} = P\{L_1(u_1) \lor N(w)\} = P\{L_1(u_1)\} + P\{N(w)\} - P\{L_1(u_1) \land N(w)\},\$$

where

$$P\{L_1(u_1) \land N(w)\} = P\{L_1(u_1)\} \cdot P\{N(w)\}$$

in the case of independent  $L_1$  and N, and

$$P\{L_1(u_1) \land N(w)\} = P\{L_1(u_1)\} \cdot P\{N(w) \mid L_1(u_1)\}$$

otherwise. Note that the predicate H(v) is not participating in the calculation of  $P_{SAF}$  directly. Instead, it defines "the time and the place" when and where the values of the variables u and v should be considered, and, as long as it specifies the hazardous situation following the stages defined by F and  $\tilde{F}$ , it can be understood as the *post-state* for all the probabilistic events.

In the next section we will demonstrate the approach presented above by a case study – an automatic railway crossing system.

# 4 Case Study

To illustrate safety analysis in the probabilistically enriched Event-B method, in this section we present a quantitative safety analysis of a radio-based railway crossing. This case study is included into priority program 1064 of the German Research Council (DFG) prepared in cooperation with Deutsche Bahn AG. The main difference between the proposed technology and traditional control systems of railway crossings is that signals and sensors on the route are replaced by radio communication and software computations performed at the train and railway crossings. Formal system modelling of such a system has been undertaken previously [16, 15]. However, the presented methodology is focused on logical (qualitative) reasoning about safety and does not include quantitative safety analysis. Below we demonstrate how to integrate formal modelling and probabilistic safety analysis.

Let us now briefly describe the functioning of a radio-based railway crossing system. The train on the route continuously computes its position. When it approaches a crossing, it broadcasts a *close* request to the crossing. When the railway crossing receives the command *close*, it performs some routine control to ensure safe train passage. It includes switching on the traffic lights, that is followed by an attempt to close the barriers. Shortly before the train reaches the *latest braking point*, i.e., the latest point where it is still possible for the train to stop safely, it requests the *status* of the railway crossing. If the crossing is secured, it responds with a *release* signal, which indicates that the train may pass the crossing. Otherwise, the train has to brake and stop before the crossing. More detailed requirements can be found in [16] for instance.

In our development we abstract away from modelling train movement, calculating train positions and routine control by the railway crossing. Let us note that, any time when the train approaches to the railway crossing, it sequentially performs a number of predefined operations:

• it sends the *close* request to the crossing controller;

- after a delay it sends the *status* request;
- it awaits for an answer from the crossing controller.

The crossing controller, upon receiving the close request, tries to close the barriers and, if successful, sends the *release* signal to the train. Otherwise, it does not send any signal and in this case the train activates the emergency brakes. Our safety analysis focused on defining the hazardous events that may happen in such a railway crossing system due to different hardware and/or communication failures, and assess the probability of the hazard occurrences. We make the following fault assumptions:

- the radio communication is unreliable and can cause messages to be lost;
- the crossing barrier motors may fail to start;
- the positioning sensors that are used by the crossing controller to determine a physical position of the barriers are unreliable;
- the train emergency brakes may fail.

**The abstract model.** We start our development with identification of all the high-level subsystems we have to model. Essentially, our system consists of two main components – the train and the crossing controller. The system environment is represented by the physical position of the train. Therefore, each control cycle consists of three main phases – Env, Train and Crossing. To indicate the current *phase* the eponymous variable is used.

The type modelling abstract train positions is defined as the enumerated set of nonnegative integers  $POS\_SET = \{0, CRP, SRP, SRS, DS\}$ , where 0 < CRP < SRP < SRS < DS. Each value of  $POS\_SET$  represents a specific position of the train. Here 0 stands for some initial train position outside the communication area, CRP and SRP stand for the close and status request points, and SRS and DS represent the safe reaction and danger spots respectively. The actual train position is modelled by the variable train\_pos  $\in POS\_SET$ . In addition, we use the boolean variable  $emrg\_brakes$  to model the status of the train emergency brakes. We assume that initially they are not triggered, i.e.,  $emrg\_brakes = FALSE$ .

The crossing has two barriers – one at each side of the crossing. The status of the barriers is modelled by the variables  $bar_1$  and  $bar_2$  that can take values *Opened* and *Closed*. We assume that both barriers are initially open.

The initial abstract machine *RailwayCrossing* is illustrated in Fig. 3. We omit showing here the *Initialisation* event and the **Invariants** clause (it merely defines the types of variables). Due to lack of space, in the rest

```
Machine RailwayCrossing
  Variables train_pos, phase, emrg_brakes, bar1, bar2
Invariants ....
Events
       UpdatePosition_1 \cong
            when
                 phase = Env \wedge train\_pos < DS \wedge emrg\_brakes = FALSE
            then
                 train_pos := min(\{p \mid p \in POS\_SET \land p > train_pos\}) || phase := Train_pos\}) || phase := Train_pos = 
            end
       UpdatePosition_2 \cong
            when
               phase = Env \land ((train_{pos} = DS \land emrg_{brakes} = FALSE) \lor emrg_{brakes} = TRUE)
            then
                 skip
            end
       TrainIdle \stackrel{\frown}{=}
            when
                 phase = Train \wedge train_{pos} \neq SRS
            then
                phase := Crossing
            end
       TrainReact \cong
            when
                 phase = Train \wedge train_{pos} = SRS
            then
                 emrg\_brakes :\in BOOL || phase := Crossing
            end
       CrossingBars \mathrel{\widehat{=}}
            when
                 phase = Crossing \wedge train\_pos = CRP
            then
                 bar_1, bar_2 :\in BAR\_POS || phase := Env
            end
       CrossingIdle \ \widehat{=} \\
            when
                phase = Crossing \wedge train_pos \neq CRP
            then
                phase := Env
            end
```

Figure 3: Railway crossing: the abstract machine

of the section we will also present only some selected excerpts of the model. The full Event-B specifications of the *Railway crossing system* can be found in the appendix.

In the machine RailwayCrossing we consider only the basic functionality of the system. Two events  $UpdatePosition_1$  and  $UpdatePosition_2$  are used to abstractly model train movement. The first event models the train movement outside the danger spot by updating the train abstract position according to the next value of the POS\_SET. The event  $UpdatePosition_2$ models the train behaviour after it has passed the last braking point or when it has stopped in the safe reaction spot. Essentially, this event represents the system termination (both safe and unsafe cases), which is modelled as infinite stuttering, i.e., keeping the system in the final state forever. Such an approach for modelling of the train movement is sufficient since we only analyse system behaviour within the train-crossing communication area, i.e., the area that consists of the close and status request points, and the safe reaction spot. A more realistic approach for modelling of the train movement is out of the scope of our safety analysis.

For the crossing controller, we abstractly model closing of the barriers by the event CrossingBar, which non-deterministically assigns the variables  $bar_1$  and  $bar_2$  from the set  $BAR\_POS$ . Let us note that in the abstract machine the crossing controller immediately knows when the train enters the close request area and makes an attempt to close the barriers. In further refinement steps we eliminate this unrealistic abstraction by introducing communication between the train and the crossing controller. In addition, in the Train phase the event TrainReact non-deterministically models triggering of the train emergency brakes in the safe reaction spot.

The hazard present in the system is the situation when the train passes the crossing while at least one barrier is not closed. In terms of the introduced system variables and their states it can defined as follows:

$$train\_pos = DS \land (bar_1 = Opened \lor bar_2 = Opened).$$

In a more traditional (for Event-B invariants) form, this hazard can be dually reformulated as the following safety property:

$$train\_pos = SRS \land phase = Crossing \Rightarrow$$
$$(bar_1 = Closed \land bar_2 = Closed) \lor emrg\_brakes = TRUE.$$
(1)

This safety requirement can be interpreted as follows: after the train, being in the safe reaction spot, has reacted on signals from the crossing controller, the system is in the safe state only when both barriers are closed or the emergency brakes are activated. Obviously, this property cannot be formulated as an Event-B invariant – it might be violated due to possible communication and/or hardware failures. Our goal is to assess the probability of violation (or preservation) of the safety property (1). To achieve this, during the refinement process, we have to unfold (1) by introducing into the specification the representation of all the system components that have an impact on the system safety. Moreover, we should establish a relationship between the variables representing these components and the abstract variables presented in (1).

**The first refinement.** In the first refinement step we examine in detail the system behaviour at the safe reaction spot – the last train position preceding the danger spot where the hazard may occur. As a result, the abstract event TrainReact is refined by three events  $TrainRelease_1$ ,  $TrainRelease_2$  and TrainStop that represent reaction of the train on the presence or absence of the release signal from the crossing controller. The first two events are used

to model the situations when the release signal has been successfully delivered or lost respectively. The last one models the situation when the release signal has not been sent due to some problems at the crossing controller side. Please note that since the events *TrainRelease*<sub>2</sub> and *TrainStop* perform the same actions, i.e., trigger the emergency brakes, they differ only in their guards.

The event CrossingStatusReq that "decides" whether to send or not to send the release signal is very abstract at this stage – it does not have any specific guards except those that define the system phase and train position. Moreover, the variable  $release\_snd$  is updated in the event body nondeterministically. To model the failures of communication and emergency brakes, we introduce two new events with *probabilistic* bodies – the events ReleaseComm and TrainDec correspondingly. For convenience, we consider communication as a part of the receiving side behaviour. Thus the release communication failure occurrence is modelled in the Train phase while the train being in the SRS position. Some key details of the Event-B machine  $RailwayCrossing\_R1$  that refines the abstract machine RailwayCrossingare shown in Fig. 4.

The presence of concrete variables representing unreliable system components in  $RailwayCrossing_R1$  allows us to formulate two safety invariants  $(saf_inv_1 \text{ and } saf_inv_2)$  that glue the abstract variable  $emrg_brakes$  participating in the safety requirement (1) with the (more) concrete variables  $release_rcv, emrg_brakes_failure, release\_snd$  and  $release\_com_failure$ .

 $saf_{inv_1}$ :  $train_{pos} = SRS \land phase = Crossing \Rightarrow (emrg_brakes = TRUE \Leftrightarrow release\_rcv = FALSE \land emrg_brakes\_failure = FALSE)$ 

 $saf_{inv_2}: train_{pos} = SRS \land phase = Crossing \Rightarrow (release\_rcv = FALSE \Leftrightarrow release\_snd = FALSE \lor release\_comm\_failure = TRUE)$ 

We split the relationship between the variables into two invariant properties just to improve the readability and make the invariants easier to understand. Obviously, since the antecedents of both invariants coincide, one can easily merge them together by replacing the variable  $release\_rcv$  in  $saf\_inv_1$  with the right hand side of the equivalence in the consequent of  $saf\_inv_1$ . Please note that the variable  $release\_snd$  corresponds to a certain combination of system actions and hence should be further unfolded during the refinement process.

The second refinement. In the second refinement step we further elaborate on the system functionality. In particular, we model the request messages that the train sends to the crossing controller, as well as sensors that read the position of the barriers. Selected excerpts from the second refinement machine  $RailwayCrossing_R2$  are shown in Fig. 5. To model sending

Machine RailwayCrossing\_R1 Variables ..., release\_snd, release\_rcv, emrg\_brakes\_failure, release\_com\_failure, ... Invariants .... Events  $TrainRelease_1 \cong$ when  $phase = Train \land train\_pos = SRS \land release\_snd = TRUE$  $release\_comm\_failure = FALSE \land deceleration = FALSE \land comm\_ct = FALSE$ then  $emrg\_brakes := FALSE || release\_rcv := TRUE || phase := Crossing$  $\mathbf{end}$  $TrainRelease_2 \cong$ when  $phase = Train \land train\_pos = SRS \land release\_snd = TRUE$  $release\_comm\_failure = TRUE \land deceleration = FALSE \land comm\_ct = FALSE$ then  $emrg\_brakes: | \ emrg\_brakes' \in BOOL \land (emrg\_brakes' = TRUE \Leftrightarrow$  $emrg_brakes_failure = FALSE$ )  $release\_rcv := TRUE || phase := Crossing$ end  $TrainStop \cong$ when  $phase = Train \wedge train\_pos = SRS \wedge release\_snd = FALSE \wedge deceleration = FALSE$ then end  $CrossingStatusReq \cong$ when  $phase = Crossing \wedge train\_pos = SRP$ then  $release\_snd :\in BOOL || phase := Env$ end  $ReleaseComm \cong$ when  $phase = Train \wedge train\_pos = SRS \wedge release\_snd = TRUE \wedge comm\_ct = TRUE$ then  $release\_comm\_failure \oplus | TRUE @ p_1; FALSE @ 1-p_1 || comm\_ct := FALSE$ end  $TrainDec \cong$ when  $phase = Train \wedge train\_pos = SRS \wedge deceleration = TRUE$ then  $emrg\_brakes\_failure \oplus | TRUE @ p_4; FALSE @ 1-p_4 || deceleration := FALSE$ end

Figure 4: Railway crossing: first refinement

of the close and status requests by the train, we refine the event *TrainIdle* by two simple events *TrainCloseReq* and *TrainStatusReq* that activate sending of the close and status requests at the corresponding stages. In the crossing controller part, we refine the event *CrossingBars* by the event *CrossingCloseReq* that sets the actuators closing the barriers in response to the close request from the train. Clearly, in the case of communication failure occurrence during the close request transmission, both barriers remain open.

Moreover, the abstract event CrossingStatusReq is refined by two events  $CrossingStatusReq_1$  and  $CrossingStatusReq_2$  to model a reaction of the crossing controller on the status request. The former event is used to model

```
Machine RailwayCrossing_R2
Variables ..., close_snd, close_rcv, status_snd, status_rcv,
                                 close\_com\_failure, status\_com\_failure, sensor_1, sensor_2 \dots
Invariants
Events
           . . .
  TrainCloseReg \cong
   when
     phase = Train \wedge train_{pos} = CRP
   then
     close\_req\_snd := TRUE || phase := Crossing
   \mathbf{end}
  CrossingCloseReg \cong
   when
     phase = Crossing \land close\_reg\_snd = TRUE \land comm\_tc = FALSE
   then
     bar_1, bar_2 : | bar'_1 \in BAR\_POS \land bar'_2 \in BAR\_POS \land
                        (close\_comm\_failure = TRUE \Rightarrow bar'_1 = Opened \land bar'_2 = Opened)
     close\_req\_rcv : |close\_req\_rcv' \in BOOL \land (close\_req\_rcv' = TRUE \Leftrightarrow
                                                               close\_comm\_failure = FALSE)
     comm\_tc := TRUE || phase := Env
   end
  CrossingStatusReq_1 \cong
   when
     phase = Crossing \land status\_req\_snd = TRUE \land close\_req\_rcv = TRUE \land
     sens\_reading = FALSE \land comm\_tc = FALSE
   then
     release\_snd :| release\_snd' \in BOOL \land (release\_snd' = TRUE \Leftrightarrow
                 status\_comm\_failure = FALSE \land sensor_1 = Closed \land sensor_2 = Closed)
     status\_req\_rcv: |status\_req\_rcv' \in BOOL \land (status\_req\_rcv' = TRUE \Leftrightarrow
                                                             status\_comm\_failure = FALSE)
     comm\_tc := TRUE || phase := Env
   \mathbf{end}
  ReadSensors \cong
   when
     phase = Crossing \land status\_reg\_snd = TRUE \land sens\_reading = TRUE
   then
     sensor_1 :\in \{bar_1, bnot(bar_1)\} || sensor_2 :\in \{bar_2, bnot(bar_2)\} || sens\_reading :=
FALSE
   end
```

Figure 5: Railway crossing: second refinement

the situation when the close request has been successfully received (at the previous stage) and the latter one models the opposite situation. Notice that in the refined event  $CrossingStatusReq_1$  the controller sends the release signal only when it has received both request signals and identified that both barriers are closed. This interconnection is reflected in the safety invariant  $saf\_inv_3$ .

```
\begin{aligned} \mathbf{saf\_inv_3}: train\_pos = SRP \land phase = Env \Rightarrow \\ (release\_snd = TRUE \Leftrightarrow close\_req\_rcv = TRUE \land \\ status\_req\_rcv = TRUE \land sensor_1 = Closed \land sensor_2 = Closed) \end{aligned}
```

Here the variables  $sensor_1$  and  $sensor_2$  represent values of the barrier positioning sensors. Let us remind that the sensors are unreliable and can return

the actual position of the barriers incorrectly. Specifically, the sensors can get stuck at their previous values or spontaneously change the values to the opposite ones. In addition, to model the communication failures, we add two new events *CloseComm* and *StatusComm*. These events are similar to the *ReleaseComm* event of the *RailwayCrossing\_R1* machine. Rather intuitive dependencies between the train requests delivery and communication failure occurrences are defined by a pair of safety invariants  $saf\_inv_4$  and  $saf\_inv_5$ .

 $saf_{inv_4}: train_pos = SRP \land phase = Env \Rightarrow$  $(status\_req\_rcv = TRUE \Leftrightarrow status\_com\_failure = FALSE)$ 

$$saf\_inv_5: train\_pos = CRP \land phase = Env \Rightarrow$$
$$(close\_req\_rcv = TRUE \Leftrightarrow close\_com\_failure = FALSE)$$

The third refinement. In the third refinement step – the Event-B machine  $RailwayCrossing\_R3$  – we refine the remaining abstract representation of components mentioned in the safety requirement (1), i.e., modelling of the barrier motors and positioning sensors. We introduce the new variables  $bar\_failure_1$ ,  $bar\_failure_2$ ,  $sensor\_failure_1$  and  $sensor\_failure_2$  to model the hardware failures. These variables are assigned probabilistically in the newly introduced events BarStatus and SensorStatus in the same way as it was done for the communication and emergency brakes failures in the first refinement. We refine CrossingCloseReq and ReadSensors events accordingly. Finally, we formulate four safety invariants  $saf\_inv_6$ ,  $saf\_inv_7$ ,  $saf\_inv_8$  and  $saf\_inv_9$  to specify the correlation between the physical position of the barriers, the sensor readings, and the hardware failures.

$$saf_{inv_6}$$
:  $train_{pos} = CRP \land phase = Env \Rightarrow (bar_1 = Closed \Leftrightarrow bar_failure_1 = FALSE \land close\_comm_failure = FALSE)$ 

$$saf_{inv_7}$$
:  $train_{pos} = CRP \land phase = Env \Rightarrow (bar_2 = Closed \Leftrightarrow bar_failure_2 = FALSE \land close\_comm_failure = FALSE)$ 

$$\begin{aligned} \mathbf{saf\_inv_8} : train\_pos = SRP \land phase = Env \Rightarrow (sensor_1 = Closed \Leftrightarrow \\ ((bar_1 = Closed \land sensor\_failure_1 = FALSE) \lor \\ (bar_1 = Opened \land sensor\_failure_1 = TRUE))) \end{aligned}$$

$$\begin{aligned} \mathbf{saf\_inv_9}: train\_pos = SRP \land phase = Env \Rightarrow (sensor_2 = Closed \Leftrightarrow \\ ((bar_2 = Closed \land sensor\_failure_2 = FALSE) \lor \\ (bar_2 = Opened \land sensor\_failure_2 = TRUE))) \end{aligned}$$

The first two invariants state that the crossing barrier can be closed (in the post-state) only when the controller has received the close request and the barrier motor has not failed to start. The second pair of invariants postulates that the positioning sensor may return the value *Closed* in two cases – when the barrier is closed and the sensor works properly, or when the barrier has got stuck while opened and the sensor misreads its position.

Once we have formulated the last four safety invariants, there is no longer any variable, in the safety property (1), that cannot be expressed via some probabilistically updated variables introduced during the refinement process. This allows us to calculate the probability  $P_{SAF}$  that (1) is preserved by the system:

$$P_{SAF} = P\{(bar_1 = Closed \land bar_2 = Closed) \lor emrg\_brakes = TRUE\} = P\{bar_1 = Closed \land bar_2 = Closed\} + P\{emrg\_brakes = TRUE\} - P\{bar_1 = Closed \land bar_2 = Closed\} \cdot P\{emra\_brakes = TRUE \mid bar_1 = Closed \land bar_2 = Closed\}.$$

Let us recall that we have idenified four different types of failures in our system – the communication failure, the failure of the barrier motor, the sensor failure and emergency brakes failure. We suppose that the probabilities of all these failures are constant and equal to  $p_1$ ,  $p_2$ ,  $p_3$  and  $p_4$  correspondingly. The first probability presented in the sum above can be trivially calculated based on the safety invariants  $saf\_inv_7$  and  $saf\_inv_8$ :

$$P\{bar_1 = Closed \land bar_2 = Closed\} = P\{bar\_failure_1 = FALSE \land bar\_failure_2 = FALSE \land close\_comm\_failure = FALSE\} = (1 - p_1) \cdot (1 - p_2)^2.$$

Indeed, both barriers are closed only when the crossing controller received the close request and none of the barrier motors has failed. The calculation of the other two probabilities is slightly more complicated. Nevertheless, they can be straightforwardly obtained using all the safety invariants defined in the model and basic rules for calculating probability. We omit the computation details due to a lack of space. The resulting probability of preservation of the safety property (1) is:

$$P_{SAF} = (1 - p_1) \cdot (1 - p_2)^2 + (1 - p_4) \cdot \left(1 - (1 - p_1)^3 \cdot (p_2 \cdot p_3 + (1 - p_2) \cdot (1 - p_3))^2\right) - (1 - p_1) \cdot (1 - p_2)^2 \cdot (1 - p_4) \cdot \left(1 - (1 - p_1)^2 \cdot (1 - p_3)^2\right).$$

Please note that  $P_{SAF}$  is defined as a function of probabilities of component failures, i.e., probabilities  $p_1, \ldots, p_4$ . Provided the numerical values of them are given, we can use the obtained formula to verify whether the system achieves the desired safety threshold.

# 5 Discussion

#### 5.1 Related Work

Formal methods are extensively used for the development and verification of safety-critical systems. In particular, the B Method and Event-B are successfully being applied for formal development of railway systems [12, 5]. A safety analysis of the formal model of a radio-based railway crossing controller has also been performed with the KIV theorem prover [16, 15]. However, the approaches for integrating formal verification and quantitative assessment are still scarce.

Usually, quantitative analysis of safety relies on probabilistic model checking techniques. For instance, in [11], the authors demonstrate how the quantitative model checker PRISM [17] can be used to evaluate system dependability attributes. The work reported in [8] presents model-based probabilistic safety assessment based on generating PRISM specifications from Simulink diagrams annotated with failure logic. A method pFMEA (probabilistic Failure Modes and Effect Analysis) also relies on the PRISM model checker to conduct quantitative analysis of safety [9]. The approach integrates the failure behaviour into the system model described in continuous time Markov chains via failure injection. In [14] the authors propose a method for probabilistic model-based safety analysis for synchronous parallel systems. It has been shown that different types of failures, in particular per-time and perdemand, can be modelled and analysed using probabilistic model checking.

However, in general the methods based on model checking aim at safety evaluation of already developed systems. They extract a model eligible for probabilistic analysis and evaluate impact of various system parameters on its safety. In our approach, we aim at providing the designers with a safetyexplicit development method. Indeed, safety analysis is essentially integrated into system development by refinement. It allows us to perform quantitative assessment of safety within proof-based verification of the system behaviour.

#### 5.2 Conclusions

In this paper we have proposed an approach to integrating quantitative safety assessment into formal system development in Event-B. The main merit of our approach is that of merging logical (qualitative) reasoning about correctness of system behaviour with probabilistic (quantitative) analysis of its safety. An application of our approach allows the designers to obtain a probability of hazard occurrence as a function over probabilities of component failures.

Essentially, our approach sets the guidelines for safety-explicit development in Event-B. We have shown how to explicitly define safety properties at different levels of refinement. The refinement process has facilitated not only correctness-preserving model transformations but also establishes a logical link between safety conditions at different levels of abstraction. It leads to deriving a logical representation of hazardous conditions. An explicit modelling of probabilities of component failures has allowed us to calculate the likelihood of hazard occurrence. The B Method and Event-B are successfully and intensively used in the development of safety-critical systems, particularly in the railway domain. We believe that our approach provides the developers with a promising solution unifying formal verification and quantitative reasoning.

In our future work we are planning to further extend the proposed approach to enable probabilistic safety assessment at the architectural level.

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# Appendix

**MACHINE** RailroadCrossing

**SEES** RailroadCrossing\_ctx

#### VARIABLES

train\_pos phase emrg\_brakes

bar\_2

bar\_1

#### **INVARIANTS**

 $inv1: train_pos \in POS\_SET$   $inv2: phase \in PHASES$   $inv3: emrg\_brakes \in BOOL$   $inv4: phase \in \{Env, Train, Crossing\}$   $inv7: bar\_2 \in BAR\_POS$   $inv6: bar\_1 \in BAR\_POS$   $inv8: phase \neq Env \Rightarrow train\_pos \neq 0$ 

#### EVENTS

#### Initialisation

#### begin

```
act1 : train_pos := 0
act2 : phase := Env
act3 : emrg_brakes := FALSE
act5 : bar_2 := Opened
act4 : bar_1 := Opened
```

#### $\mathbf{end}$

**Event**  $UpdatePos1 \cong$ 

when

```
grd1 : phase = Env
grd2 : emrg_brakes = FALSE
grd3 : train_pos < DS</pre>
```

#### $\mathbf{then}$

```
act1: train_pos := min(\{p | p \in POS\_SET \land p > train\_pos\})
act2: phase := Train
```

#### $\mathbf{end}$

**Event**  $UpdatePos2 \cong$ 

#### when

#### $\mathbf{then}$

skip

#### $\mathbf{end}$

**Event** TrainIdle  $\hat{=}$ 

#### when

grd1: phase = Traingrd2:  $train_pos \neq SRS$ 

#### then

act1: phase := Crossing

#### $\mathbf{end}$

#### 

#### when

grd1: phase = Train
grd2: train\_pos = SRS

#### then

act1:  $emrg\_brakes :\in BOOL$ act2: phase := Crossing

#### end

**Event** CrossingBars  $\hat{=}$ 

#### when

grd1 : phase = Crossing
grd2 : train\_pos = CRP

#### then

act1:  $bar_1 :\in BAR\_POS$ act2:  $bar_2 :\in BAR\_POS$ 

```
act3: phase := Env
```

 $\mathbf{end}$ 

```
Event CrossingIdle \cong

when

grd1: phase = Crossing

grd2: train_pos \neq CRP

then

act1: phase := Env

end
```

END

 ${\bf CONTEXT} \quad {\rm RailroadCrossing\_ctx} \\$ 

SETS

PHASES

BAR\_POS

#### CONSTANTS

CRP	Close Request Period
SRP	Status Request Period
SRS	Safe Reaction Spot
DS	Danger Spot
POS_SET	
Env	Environment (train movement)
Train	Train controller/actuators phase
Crossing Crossing controller/actuators phase	
Opene	d

Closed

bnot

# AXIOMS

$\texttt{axm1}: CRP \in \mathbb{N}_1$
$\texttt{axm2}: SRP \in \mathbb{N}_1$
$\texttt{axm3}: SRS \in \mathbb{N}_1$
$\mathtt{axm4}: \ DS \in \mathbb{N}_1$
axm6: CRP < SRP
axm7: SRP < SRS
axm8: SRS < DS
$\texttt{axm10}: partition(PHASES, \{Env\}, \{Train\}, \{Crossing\})$
$\texttt{axm11}: partition(BAR\_POS, \{Opened\}, \{Closed\})$
$\texttt{axm13}: \ POS\_SET \subseteq \mathbb{N}$
$\texttt{axm12}: POS\_SET = \{0, CRP, SRP, SRS, DS\}$
$\texttt{axm14}: bnot \in BAR\_POS \rightarrow BAR\_POS$
axm15: bnot(Opened) = Closed
axm16: bnot(Closed) = Opened
$\texttt{thm1}: \ \forall x \cdot x \in BAR\_POS \Rightarrow (\{x, bnot(x)\} = BAR\_POS)$

END

MACHINE RailroadCrossing\_R1

**REFINES** RailroadCrossing

**SEES** RailroadCrossing\_ctx

#### VARIABLES

train\_pos
phase
emrg\_brakes
bar\_2
bar\_1
release\_snd
release\_rcv
communication\_ct
deceleration
emrg\_brakes\_failure

release\_comm\_failure

#### **INVARIANTS**

inv1: release\_snd  $\in BOOL$ 

- inv2: release\_rcv  $\in BOOL$
- inv3:  $communication\_ct \in BOOL$
- inv4: deceleration  $\in BOOL$
- inv5: emrg\_brakes\_failure  $\in BOOL$
- inv6:  $release\_comm\_failure \in BOOL$
- inv7:  $train_pos = SRS \land phase = Crossing \Rightarrow (emrg_brakes = TRUE \Leftrightarrow release\_rcv = FALSE \land emrg_brakes_failure = FALSE)$
- $inv8: train_pos = SRS \land phase = Crossing \Rightarrow (release\_rcv = FALSE \Leftrightarrow release\_snd = FALSE \lor release\_comm\_failure = TRUE)$

#### EVENTS

#### Initialisation

extended

#### begin

act1 : train\_pos := 0
act2 : phase := Env
act3 : emrg\_brakes := FALSE

act5: bar\_2 := Opened
act4: bar\_1 := Opened
act6: release\_snd := FALSE
act7: release\_rcv := FALSE
act8: communication\_ct := TRUE
act9: deceleration := TRUE
act10: emrg\_brakes\_failure := FALSE
act11: release\_comm\_failure := FALSE

#### $\mathbf{end}$

**Event**  $UpdatePos1 \cong$ 

**extends** UpdatePos1

#### when

grd1 : phase = Env
grd2 : emrg\_brakes = FALSE
grd3 : train\_pos < DS</pre>

#### $\mathbf{then}$

```
\label{eq:act1:train_pos} \begin{array}{l} \texttt{act1:train_pos} := \texttt{min}(\{p | p \in \texttt{POS\_SET} \land p > \texttt{train\_pos}\}) \\ \texttt{act2:phase} := \texttt{Train} \end{array}
```

#### $\mathbf{end}$

**Event**  $UpdatePos2 \cong$ 

**extends** UpdatePos2

#### when

```
grd1 : phase = Env
grd2 : (emrg_brakes = FALSE \land train_pos = DS) \lor emrg_brakes = TRUE
```

then

skip

#### $\mathbf{end}$

**Event** TrainIdle  $\hat{=}$ 

extends TrainIdle

when

```
\texttt{grd1}: \texttt{phase} = \texttt{Train}
\texttt{grd2}: \texttt{train\_pos} \neq \texttt{SRS}
```

then

act1 : phase := Crossing

end

**Event** TrainRelease1  $\hat{=}$ 

**refines** TrainReact

#### when

grd1 : phase = Train grd2 : train\_pos = SRS grd3 : release\_comm\_failure = FALSE grd4 : release\_snd = TRUE grd5 : communication\_ct = FALSE grd6 : deceleration = FALSE

#### then

act1 : emrg\_brakes := FALSE
act2 : phase := Crossing
act3 : release\_rcv := TRUE

#### $\mathbf{end}$

**Event** TrainRelease  $2 \cong$ 

refines TrainReact

#### when

grd1 : phase = Train grd2 : train\_pos = SRS grd3 : release\_comm\_failure = TRUE grd4 : release\_snd = TRUE grd5 : communication\_ct = FALSE grd6 : deceleration = FALSE

#### $\mathbf{then}$

#### end

**Event** TrainStop  $\hat{=}$ 

**refines** TrainReact

when

grd1: phase = Train $grd2: train_pos = SRS$  $grd3: release\_snd = FALSE$ grd4: deceleration = FALSE

#### then

```
act1: emrg\_brakes: |emrg\_brakes' \in BOOL \land (emrg\_brakes' = TRUE \Leftrightarrow
     emrg\_brakes\_failure = FALSE)
act2: phase := Crossing
act3: release\_rcv := FALSE
```

#### end

```
Event CrossingBars \hat{=}
```

**extends** CrossingBars

#### when

grd1 : phase = Crossing grd2 : train\_pos = CRP then act1 : bar\_1 :∈ BAR\_POS

act2: bar\_2:∈ BAR\_POS act3 : phase := Env

#### end

**Event**  $CrossingStatusReq \cong$ 

**refines** CrossingIdle

#### when

grd1: phase = Crossing $grd2: train_pos = SRP$ 

#### then

act1:  $release\_snd :\in BOOL$ act2: phase := Env

#### end

**Event** CrossingIdle  $\hat{=}$ 

**refines** CrossingIdle

#### when

grd1: phase = Crossing $grd2: train_pos \in \{SRS, DS\}$ 

#### then

act1: phase := Envact2:  $release\_snd := FALSE$ 

#### $\mathbf{end}$

**Event** Release Comm  $\hat{=}$ 

#### when

grd1 : phase = Train
grd2 : train\_pos = SRS
grd3 : release\_snd = TRUE
grd4 : communication\_ct = TRUE

#### $\mathbf{then}$

act1:  $release\_comm\_failure :\in BOOL$ act2:  $communication\_ct := FALSE$ 

#### end

**Event**  $TrainDec \cong$ 

#### $\mathbf{when}$

grd1 : phase = Train
grd2 : train\_pos = SRS
grd3 : deceleration = TRUE

#### then

act1:  $emrg\_brakes\_failure :\in BOOL$ act2: deceleration := FALSE

#### end

END

MACHINE RailroadCrossing\_R2

**REFINES** RailroadCrossing\_R1

**SEES** RailroadCrossing\_ctx

#### VARIABLES

train\_pos phase emrg\_brakes bar\_2 bar\_1 release\_snd release\_rcv communication\_ct deceleration emrg\_brakes\_failure release\_comm\_failure close\_req\_snd close\_req\_rcv status\_req\_snd status\_req\_rcv sens\_reading sensor\_1 sensor\_2 communication\_tc close\_comm\_failure status\_comm\_failure

#### **INVARIANTS**

inv1:  $close\_req\_snd \in BOOL$ inv2:  $close\_req\_rcv \in BOOL$ inv3:  $status\_req\_snd \in BOOL$ inv4:  $status\_req\_rcv \in BOOL$ inv5:  $sens\_reading \in BOOL$ inv6:  $sensor\_1 \in BAR\_POS$ inv7:  $sensor\_2 \in BAR\_POS$  inv8: communication\_tc  $\in BOOL$ 

 $inv9: close\_comm\_failure \in BOOL$ 

- inv10:  $status\_comm\_failure \in BOOL$
- $inv12: train_pos = SRP \land phase = Env \Rightarrow (status_req_rcv = TRUE \Leftrightarrow status_comm_failure = FALSE)$
- inv13:  $train_pos = CRP \land phase = Env \Rightarrow (close_req_rcv = TRUE \Leftrightarrow close_comm_failure = FALSE)$
- inv14:  $phase = Crossing \Rightarrow (close\_req\_snd = TRUE \Leftrightarrow train\_pos = CRP)$
- inv15:  $phase = Crossing \Rightarrow (status\_req\_snd = TRUE \Leftrightarrow train\_pos = SRP)$
- inv16:  $close\_req\_snd = TRUE \Rightarrow status\_req\_snd = FALSE$
- inv17: train\_pos < SRP  $\Rightarrow$  status\_req\_snd = FALSE
- inv18:  $train_pos > SRP \Rightarrow close_req\_snd = FALSE$
- inv19:  $train_{pos} = SRP \land phase = Env \Rightarrow close_{req_snd} = FALSE$
- inv20:  $train_pos = DS \Rightarrow status_req\_snd = FALSE$
- inv21:  $train_pos = SRS \land phase = Env \Rightarrow status_req_snd = FALSE$

#### EVENTS

#### Initialisation

extended

#### begin

act1: train\_pos := 0 act2: phase := Env act3: emrg\_brakes := FALSE act5: bar\_2 := Opened act4: bar\_1 := Opened act6: release\_snd := FALSE act7: release\_rcv := FALSE act8: communication\_ct := TRUE act9: deceleration := TRUE act10: emrg\_brakes\_failure := FALSE act11: release\_comm\_failure := FALSE act12: close\_req\_snd := FALSE act13: close\_req\_rcv := FALSE act14: status\_req\_snd := FALSE

```
act15: status_req_rcv := FALSE
act16: sens_reading := TRUE
act17: sensor_1 := Opened
act18: sensor_2 := Opened
act19: communication_tc := TRUE
act20: close_comm_failure := FALSE
act21: status_comm_failure := FALSE
```

#### $\mathbf{end}$

**Event**  $UpdatePos1 \cong$ 

```
extends UpdatePos1
```

#### when

```
grd1 : phase = Env
grd2 : emrg_brakes = FALSE
grd3 : train_pos < DS</pre>
```

#### $\mathbf{then}$

```
\label{eq:act1} \begin{array}{ll} \texttt{act1}: \texttt{train_pos} := \texttt{min}(\{p | p \in \texttt{POS\_SET} \land p > \texttt{train_pos}\}) \\ \texttt{act2}: \texttt{phase} := \texttt{Train} \end{array}
```

#### $\mathbf{end}$

**Event**  $UpdatePos2 \cong$ 

**extends** UpdatePos2

#### when

```
grd1 : phase = Env
grd2 : (emrg_brakes = FALSE \land train_pos = DS) \lor emrg_brakes = TRUE
```

then

skip

end

**Event**  $TrainCloseReq \cong$ 

refines TrainIdle

when

grd1 : phase = Train
grd2 : train\_pos = CRP

#### then

act1: phase := Crossing

act2:  $close\_req\_snd := TRUE$ 

end

```
Event TrainStatusReq \cong
```

#### **refines** TrainIdle

#### when

grd1: phase = Train
grd2: train\_pos = SRP

#### then

act1: status\_req\_snd := TRUE
act2: close\_req\_snd := FALSE
act3: phase := Crossing

#### end

**Event**  $TrainRelease1 \cong$ 

**extends** TrainRelease1

#### when

grd1:	$\mathtt{phase} = \mathtt{Train}$
grd2:	$\texttt{train\_pos} = \texttt{SRS}$
grd3:	$\texttt{release\_comm\_failure} = \texttt{FALSE}$
grd4:	$\texttt{release\_snd} = \texttt{TRUE}$
grd5:	$\texttt{communication\_ct} = \texttt{FALSE}$
grd6:	deceleration = FALSE

#### then

```
act1 : emrg_brakes := FALSE
act2 : phase := Crossing
act3 : release_rcv := TRUE
act4 : status_req_snd := FALSE
```

#### $\mathbf{end}$

**Event**  $TrainRelease2 \cong$ 

**extends** TrainRelease2

#### when

```
grd1 : phase = Train
grd2 : train_pos = SRS
grd3 : release_comm_failure = TRUE
grd4 : release_snd = TRUE
```

```
grd5 : communication_ct = FALSE
grd6 : deceleration = FALSE
```

#### then

```
act1: emrg_brakes: |emrg_brakes' ∈ BOOL ∧ (emrg_brakes' = TRUE ⇔
    emrg_brakes_failure = FALSE)
act2: phase := Crossing
act3: release_rcv := FALSE
act4: status_req_snd := FALSE
```

#### end

```
Event TrainStop \hat{=}
```

```
extends TrainStop
```

#### when

grd1 : phase = Train grd2 : train\_pos = SRS grd3 : release\_snd = FALSE grd4 : deceleration = FALSE

#### $\mathbf{then}$

```
act1 : emrg_brakes : |emrg_brakes' ∈ BOOL∧(emrg_brakes' = TRUE⇔
emrg_brakes_failure = FALSE)
act2 : phase := Crossing
act3 : release_rcv := FALSE
act4 : status_req_snd := FALSE
```

#### end

```
Event TrainDangerSpot \hat{=}
```

**refines** TrainIdle

#### when

grd1 : phase = Train
grd2 : train\_pos = DS

#### then

act1: phase := Crossing

#### end

**Event**  $CrossingCloseReq \cong$ 

refines CrossingBars

when

grd1 : phase = Crossing
grd2 : close\_req\_snd = TRUE
grd3 : communication\_tc = FALSE

#### then

```
act1: bar_1, bar_2: |bar_1' \in BAR\_POS \land bar_2' \in BAR\_POS \land (close\_comm\_failure = TRUE \Rightarrow bar_1' = Opened \land bar_2' = Opened)act3: close\_req\_rcv: |close\_req\_rcv' \in BOOL \land (close\_req\_rcv' = TRUE \Leftrightarrow close\_comm\_failure = FALSE)act4: communication\_tc := TRUEact5: phase := Env
```

#### $\mathbf{end}$

**Event** CrossingStatusReq1  $\hat{=}$ 

**refines** CrossingStatusReq

#### when

grd1 : phase = Crossing grd2 : status\_req\_snd = TRUE grd3 : close\_req\_rcv = TRUE grd4 : sens\_reading = FALSE grd5 : communication\_tc = FALSE

#### then

```
act2: status\_req\_rcv : |status\_req\_rcv' \in BOOL \land (status\_req\_rcv' = TRUE \Leftrightarrow status\_comm\_failure = FALSE)
```

act3:  $communication\_tc := TRUE$ 

act4: phase := Env

#### end

**Event** CrossingStatusReq2  $\hat{=}$ 

**refines** CrossingStatusReq

#### when

grd1 : phase = Crossing grd2 : status\_req\_snd = TRUE grd3 : close\_req\_rcv = FALSE grd4 : sens\_reading = FALSE  $grd5: communication_tc = FALSE$ 

#### then

```
\begin{array}{l} \texttt{act1}: \ status\_req\_rcv: \ | \ status\_req\_rcv' \in BOOL \land (status\_req\_rcv' = TRUE \Leftrightarrow status\_comm\_failure = FALSE) \\ \texttt{act2}: \ release\_snd := FALSE \\ \texttt{act3}: \ communication\_tc := TRUE \\ \texttt{act4}: \ phase := Env \end{array}
```

#### end

**Event** CrossingIdle  $\hat{=}$ 

#### **refines** CrossingIdle

#### when

grd1 : phase = Crossing
grd2 : close\_req\_snd = FALSE
grd3 : status\_req\_snd = FALSE

#### then

act1 : phase := Env
act2 : release\_snd := FALSE

#### $\mathbf{end}$

**Event** Release Comm  $\hat{=}$ 

#### **extends** ReleaseComm

#### when

grd1 : phase = Train
grd2 : train\_pos = SRS
grd3 : release\_snd = TRUE
grd4 : communication\_ct = TRUE

#### then

 $\texttt{act1}: \texttt{release\_comm\_failure} :\in \texttt{BOOL}$ 

```
\verb+act2: communication\_ct:= FALSE
```

#### end

extends TrainDec

#### when

```
grd1 : phase = Train
grd2 : train_pos = SRS
```

```
grd3 : deceleration = TRUE
```

#### then

#### $\mathbf{end}$

**Event**  $CloseComm \cong$ 

#### $\mathbf{when}$

grd1 : phase = Crossing grd2 : close\_req\_snd = TRUE grd3 : communication\_tc = TRUE

#### then

act1:  $close\_comm\_failure :\in BOOL$ act2:  $communication\_tc := FALSE$ 

#### end

**Event** StatusComm  $\hat{=}$ 

#### when

grd1 : phase = Crossing grd2 : status\_req\_snd = TRUE grd3 : communication\_tc = TRUE

#### then

act1:  $status\_comm\_failure :\in BOOL$ act2:  $communication\_tc := FALSE$ 

#### end

**Event** ReadSensors  $\hat{=}$ 

#### when

grd1 : phase = Crossing
grd2 : status\_req\_snd = TRUE
grd3 : sens\_reading = TRUE

#### then

 $\mathbf{end}$ 

 $\mathbf{END}$ 

MACHINE RailwayCrossing\_R3

**REFINES** RailroadCrossing\_R2

**SEES** RailroadCrossing\_ctx

#### VARIABLES

train\_pos phase emrg\_brakes bar\_2 bar\_1 release\_snd release\_rcv communication\_ct deceleration emrg\_brakes\_failure release\_comm\_failure close\_req\_snd close\_req\_rcv status\_req\_snd status\_req\_rcv sens\_reading sensor\_1 sensor\_2 communication\_tc close\_comm\_failure status\_comm\_failure bar\_failure\_1 bar\_failure\_2 sensor\_failure\_1 sensor\_failure\_2 closing sensing

#### **INVARIANTS**

inv1: bar\_failure\_1  $\in BOOL$ 

- $inv2: bar_failure_2 \in BOOL$
- inv3: sensor\_failure\_1  $\in BOOL$
- inv4: sensor\_failure\_2  $\in BOOL$
- inv5:  $closing \in BOOL$
- **inv6**:  $sensing \in BOOL$
- $\begin{array}{ll} \texttt{inv7}: \ train\_pos = CRP \land phase = Env \Rightarrow (bar\_1 = Closed \Leftrightarrow bar\_failure\_1 = FALSE \land close\_comm\_failure = FALSE) \land (bar\_2 = Closed \Leftrightarrow bar\_failure\_2 = FALSE \land close\_comm\_failure = FALSE) \end{array}$
- $\begin{array}{l} \texttt{inv9}: \ sens\_reading = FALSE \Rightarrow (sensor\_1 = Closed \Leftrightarrow ((bar\_1 = Closed \land sensor\_failure\_1 = FALSE) \lor (bar\_1 = Opened \land sensor\_failure\_1 = TRUE))) \land (sensor\_2 = Closed \Leftrightarrow ((bar\_2 = Closed \land sensor\_failure\_2 = FALSE) \lor (bar\_2 = Opened \land sensor\_failure\_2 = TRUE))) \end{array}$
- inv10:  $sensing = TRUE \Rightarrow sens\_reading = TRUE$

inv11:  $sens\_reading = FALSE \Rightarrow train\_pos > CRP$ 

#### EVENTS

#### Initialisation

extended

#### begin

```
act1 : train_pos := 0
act2 : phase := Env
act3 : emrg_brakes := FALSE
act5 : bar_2 := Opened
act4 : bar_1 := Opened
act6 : release_snd := FALSE
act7 : release_rcv := FALSE
act8 : communication_ct := TRUE
act9 : deceleration := TRUE
act10 : emrg_brakes_failure := FALSE
act11 : release_comm_failure := FALSE
act12 : close_req_snd := FALSE
act13 : close_req_rcv := FALSE
act14 : status_req_snd := FALSE
act15 : status_req_rcv := FALSE
act16 : sens_reading := TRUE
```

act17: sensor\_1 := Opened act18: sensor\_2 := Opened act19: communication\_tc := TRUE act20: close\_comm\_failure := FALSE act21: status\_comm\_failure := FALSE act22: bar\_failure\_1 := FALSE act23: bar\_failure\_2 := FALSE act24: sensor\_failure\_1 := FALSE act25: sensor\_failure\_2 := FALSE act26: closing := TRUE act27: sensing := TRUE

#### end

**Event**  $UpdatePos1 \cong$ 

**extends** UpdatePos1

#### when

grd1 : phase = Env
grd2 : emrg\_brakes = FALSE
grd3 : train\_pos < DS</pre>

#### then

```
\label{eq:act1:train_pos} \begin{array}{l} \texttt{act1:train_pos} := \texttt{min}(\{p | p \in \texttt{POS\_SET} \land p > \texttt{train\_pos}\}) \\ \texttt{act2:phase} := \texttt{Train} \end{array}
```

#### $\mathbf{end}$

**Event**  $UpdatePos2 \cong$ 

**extends** UpdatePos2

when

```
grd1 : phase = Env
grd2 : (emrg_brakes = FALSE \land train_pos = DS) \lor emrg_brakes = TRUE
```

then

skip

#### end

**Event**  $TrainCloseReq \cong$ 

**extends** TrainCloseReq

when

```
grd1 : phase = Train
grd2 : train_pos = CRP
then
act1 : phase := Crossing
act2 : close_req_snd := TRUE
end
```

**Event**  $TrainStatusReq \cong$ 

 ${\bf extends} \ \ TrainStatusReq$ 

#### when

grd1 : phase = Train
grd2 : train\_pos = SRP

#### $\mathbf{then}$

act1 : status\_req\_snd := TRUE
act2 : close\_req\_snd := FALSE
act3 : phase := Crossing

#### end

**Event** TrainRelease1  $\hat{=}$ 

**extends** TrainRelease1

#### when

grd1: phase = Train grd2: train\_pos = SRS grd3: release\_comm\_failure = FALSE grd4: release\_snd = TRUE grd5: communication\_ct = FALSE grd6: deceleration = FALSE

#### then

act1 : emrg\_brakes := FALSE act2 : phase := Crossing act3 : release\_rcv := TRUE act4 : status\_req\_snd := FALSE

#### end

**Event**  $TrainRelease2 \cong$ **extends** TrainRelease2

when

```
grd1: phase = Train
grd2: train_pos = SRS
grd3: release_comm_failure = TRUE
grd4: release_snd = TRUE
grd5: communication_ct = FALSE
grd6: deceleration = FALSE
```

#### then

```
act1: emrg_brakes: |emrg_brakes' ∈ BOOL ∧ (emrg_brakes' = TRUE ⇔
    emrg_brakes_failure = FALSE)
act2: phase := Crossing
act3: release_rcv := FALSE
act4: status_req_snd := FALSE
```

#### $\mathbf{end}$

**Event** TrainStop  $\hat{=}$ 

extends TrainStop

#### when

grd1:	$\mathtt{phase} = \mathtt{Train}$
grd2:	$\texttt{train\_pos} = \texttt{SRS}$
grd3:	$\texttt{release\_snd} = \texttt{FALSE}$
grd4:	deceleration = FALSE

#### then

```
act1 : emrg_brakes : |emrg_brakes' ∈ BOOL ∧ (emrg_brakes' = TRUE ⇔
    emrg_brakes_failure = FALSE)
act2 : phase := Crossing
act3 : release_rcv := FALSE
act4 : status_req_snd := FALSE
```

#### end

**Event** TrainDangerSpot  $\hat{=}$ 

extends TrainDangerSpot

#### when

grd1 : phase = Train
grd2 : train\_pos = DS

then

act1 : phase := Crossing

end

**Event** CrossingCloseReq1  $\hat{=}$ 

**refines** CrossingCloseReq

#### when

grd1 : phase = Crossing grd2 : close\_req\_snd = TRUE grd3 : communication\_tc = FALSE grd4 : closing = FALSE grd5 : close\_comm\_failure = FALSE

#### then

#### end

**Event** CrossingCloseReq2  $\hat{=}$ 

**refines** CrossingCloseReq

#### when

grd1 : phase = Crossing grd2 : close\_req\_snd = TRUE grd3 : communication\_tc = FALSE grd4 : closing = FALSE grd5 : close\_comm\_failure = TRUE

#### $\mathbf{then}$

act1: bar\_1, bar\_2 := Opened, Opened
act2: close\_req\_rcv := FALSE
act3: communication\_tc := TRUE
act4: phase := Env

#### $\mathbf{end}$

**Event** CrossingStatusReq1  $\hat{=}$ 

**extends** CrossingStatusReq1

#### when

grd1 : phase = Crossing

grd2: status\_req\_snd = TRUE
grd3: close\_req\_rcv = TRUE
grd4: sens\_reading = FALSE
grd5: communication\_tc = FALSE

#### $\mathbf{then}$

```
act1 : release_snd : |release_snd' ∈ BOOL∧(release_snd' = TRUE⇔
    (status_comm_failure = FALSE∧sensor_1 = Closed∧sensor_2 =
    Closed))
act2 : status_req_rcv : |status_req_rcv' ∈ BOOL∧(status_req_rcv' =
    TRUE ⇔ status_comm_failure = FALSE)
act3 : communication_tc := TRUE
act4 : phase := Env
```

#### end

**Event** CrossingStatusReq2  $\hat{=}$ 

**extends** CrossingStatusReq2

#### when

grd1 : phase = Crossing grd2 : status\_req\_snd = TRUE grd3 : close\_req\_rcv = FALSE grd4 : sens\_reading = FALSE grd5 : communication\_tc = FALSE

#### then

```
act1: status_req_rcv: |status_req_rcv' ∈ BOOL∧(status_req_rcv' =
    TRUE ⇔ status_comm_failure = FALSE)
act2: release_snd := FALSE
act3: communication_tc := TRUE
act4: phase := Env
```

#### $\mathbf{end}$

**Event** CrossingIdle  $\hat{=}$ 

**extends** CrossingIdle

#### when

grd1 : phase = Crossing grd2 : close\_req\_snd = FALSE grd3 : status\_req\_snd = FALSE

#### $\mathbf{then}$

act1 : phase := Env

 $act2: release\_snd := FALSE$ 

 $\mathbf{end}$ 

```
Event Release Comm \hat{=}
```

**extends** ReleaseComm

#### when

grd1: phase = Train
grd2: train\_pos = SRS
grd3: release\_snd = TRUE
grd4: communication\_ct = TRUE

#### $\mathbf{then}$

```
act1 : release_comm_failure :∈ BOOL
act2 : communication_ct := FALSE
```

#### end

**Event**  $TrainDec \cong$ 

 $\mathbf{extends} \ \ TrainDec$ 

#### when

grd1 : phase = Train
grd2 : train\_pos = SRS
grd3 : deceleration = TRUE

#### $\mathbf{then}$

act1 : emrg\_brakes\_failure :∈ BOOL
act2 : deceleration := FALSE

#### end

#### **extends** CloseComm

#### when

grd1 : phase = Crossing grd2 : close\_req\_snd = TRUE grd3 : communication\_tc = TRUE

#### then

```
act1 : close_comm_failure :∈ BOOL
act2 : communication_tc := FALSE
```

#### end

**Event** StatusComm  $\hat{=}$ 

**extends** StatusComm

#### $\mathbf{when}$

grd1 : phase = Crossing grd2 : status\_req\_snd = TRUE grd3 : communication\_tc = TRUE then

# act1 : status\_comm\_failure :∈ BOOL act2 : communication\_tc := FALSE

#### end

**Event** ReadSensors  $\hat{=}$ 

#### **refines** ReadSensors

#### when

grd1 : phase = Crossing grd2 : status\_req\_snd = TRUE grd3 : sens\_reading = TRUE grd4 : sensing = FALSE

#### then

#### end

**Event**  $BarStatus \cong$ 

#### when

grd1 : phase = Crossing
grd2 : close\_req\_snd = TRUE
grd3 : closing = TRUE

#### then

act1:  $bar_failure_1 :\in BOOL$ act2:  $bar_failure_2 :\in BOOL$ act3: closing := FALSE

#### $\mathbf{end}$

```
{\bf Event} \quad {\it SensorStatus} \ \widehat{=} \\
```

#### when

grd1 : phase = Crossing
grd2 : status\_req\_snd = TRUE
grd3 : sensing = TRUE

#### then

act1:  $sensor_failure_1 :\in BOOL$ act2:  $sensor_failure_2 :\in BOOL$ act3: sensing := FALSE

#### $\mathbf{end}$

#### $\mathbf{END}$



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